Supply Chain Finance and Firm Capital Structure*

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Abstract

We analyze a proprietary dataset of a prominent Italian banking group and provide evidence that factoring extends the typical liquidity insurance service offered by trade credit to customers. It also alleviates financial constraints for suppliers. In addition, suppliers use factoring to transfer surplus to their high-bargaining customers, and to broaden their customer base. To align with these observations, we construct a structural model where the utilization of factoring is endogenously determined, along with the capital structure of the bank, supplier, and customer. Notably, due to factoring services, the structure of the network of trade credit relationships influences the aggregate bank credit portfolio payoff and riskiness. When we calibrate the model to our dataset, we show that the features of the production-related supply-chain network and downstream competition simultaneously influence inter-firm trade, bank-related debt chains, and firm-specific corporate financial decisions.

JEL codes: G2, G21, G32, G38
Keywords: Banks, capital structure, factoring, firm volatility, supply chain of credit.

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1 Introduction

In a frictionless world with perfect capital markets, optimal financial and production decisions of firms are independent. Yet, the extensive reliance of small and medium-sized enterprises on supply-chain finance solutions provides a prominent example of the shortcomings of this modeling approach. Financial frictions play a crucial role in determining firm capital structure decisions, leading to the emergence of trade-related financial connections and specialized financial intermediaries.

This study delves into the interplay between a firm's production decision and the financial relationships formed when a supplier and a customer engage in a factoring agreement. This trade-related contract is distinct for its inclusion of a financial intermediary, typically a bank acting as a factor, that provides cash or financing to companies by purchasing their accounts receivables at a discount. Factoring can support the supplier's immediate working capital requirements. To our knowledge, our paper is the first that analyzes quantitatively and qualitatively the implications of the contractual arrangement that establishes a financial supply chain where the bank's debt and credit decisions, the optimal capital structure of the firm, and the utilization of factoring services are interdependent.

Building upon the 'supply chain of credit' model by Gornall and Strebulaev [2018], our research explores the strategic interaction among stakeholders. This model introduces a fundamental asymmetry between banks borrowing capital directly and downstream borrowers benefiting from the debt. We extend this framework in three ways. Firstly, we relax the assumption that only systematic shocks affecting all firms are relevant. The structure of the trade credit network significantly influences the bank's aggregate payoff and riskiness. Secondly, we consider the differential specialization of customers and suppliers, accounting for shocks propagated through the trade credit channel. Thirdly, our model includes a financially constrained supplier and two customers with varying bargaining power competing in the downstream market.

Our modeling approach relies on empirical insights derived from the analysis of an exclusive dataset of factoring transactions extracted from a big data infrastructure of a leading Italian bank group. This dataset encompasses all bank's information on corporate customers, mainly Italian firms of different sizes, including small- and medium-sized firms within the same production chain, with limited access to capital markets. We collect data on the capital structure of both suppliers and customers within the bank's portfolio and identify the network formed by customers and suppliers involved in factoring relationships mediated by a bank subsidiary.

Applying a community detection algorithm to this empirical network unveils a distinct granular pattern. Most firms are grouped in star-like sub-networks, featuring a central supplier connected to varying numbers of customers. Notably, descriptive statistics of these clusters mirror key characteristics of the customer-supplier network analyzed in Herskovic et al. [2020], highlighting that central suppliers in these star networks aggregate shocks experienced by their connected customers.

We investigate the correlation between the frequency of trade-credit factoring agreements and the determinants of customers' and suppliers' capital structures through a regression analysis. Our findings reveal that smaller-sized firms exhibit a higher proportional use of factoring. Suppliers, in particular, employ factoring to secure short-term financing for operational expenses and financial commitments. As expected, empirical evidence suggests that factoring serves as a stabilizing force in trade-credit relationships by alleviating financial constraints for suppliers.

To delve deeper into the interplay between these constraints and the spontaneous formation of connections, we adopt a strategy similar to Giannetti et al. [2021]. Leveraging the variation induced by an exogenous shock to the supply of factoring services, specifically a reform favoring the securitization of receivables, we analyze the effects. The reorganization of the factoring network, spurred by improved credit conditions, leads suppliers to increase trade credit factoring for financially sound larger customers and new, smaller, and safer customers. Financially distressed suppliers, on the other hand, extend trade credit factoring to new, larger customers, revealing that firms utilize trade credit to transfer surplus to customers and expand their customer base.

A simple calibration of our model aligns with most empirical findings. The structural model, solved through computationally intensive numerical iterations, uncovers several key insights. First, factoring services act as an additional channel for high-market-share lenders to provide liquidity to customers and suppliers, serving as a stabilizer beyond the typical liquidity insurance offered by trade credit, documented in Cuñat [2007], and Cuñat and García-Appendini [2012]. Second, our evidence supports the findings of Giannetti and Saidi [2019] on the granular volatility of firm networks and provides a micro-foundation for granular volatility in the context of supply chain disruptions. Notably, as competition in downstream markets eases, the optimal trade credit allocation policy reduces the Herfindahl concentration index within the customer network, thereby mitigating the asset volatility of the supplier. Third, our model generates benchmark 'fair' prices for bank credit and factoring services, reproducing observed stylized facts [1], such as the significant influence of factoring services on contract valuation.

These outcomes are directly derived from the optimal risk-sharing scheme within the firm network. Additionally, as a robustness check, our calibrated model aligns with several stylized facts and replicates additional key relationships identified in the empirical analysis. For instance, the use of factoring increases with financial constraints for both customers and suppliers and equity capital acts as a buffer for highly leveraged and credit-constrained suppliers. Notably, in the presence of competing customers, the supplier's optimal trade credit allocation policy corresponds to the emergence of a scale-free distribution of network weights observed in the data.

RELATED LITERATURE. Our paper relates to and lies at the intersection of a number of different strands of literature.

[1] See for example, Amberg et al. [2021a]

First, it relates to the class of models that analyze jointly the choice of capital structure and the supply-chain relations of firms. The observation that corporate financing and operating cash flow or investment decisions are made simultaneously goes back to Myers [1974]. Harris and Raviv [1991] suggest that incorporating features of industrial organization theory into the capital structure theory has the potential to yield interesting results. Despite these long-standing observations, little has been done to theoretically explore the interaction of customer-supplier relationships and capital structures. Notable exceptions are Chu [2012], Chu and Wang [2017], and Chen et al. [2022]. Chu [2012] and Chu and Wang [2017] are mainly interested in testing two competing static theories of capital structure: the bargaining theory, and the relation-specific investment theory.[2] Chen et al. [2022] analyze the interaction between firms' reliance on a linear supply-chain and their choice of capital structure via the suppliers' product pricing. These approaches do not take into account the key role played by the financial intermediaries in providing financial support to small and medium enterprises, in particular through a factoring service.

Our analysis builds, also, on a consolidated literature that investigates the incentives that drive suppliers' willingness to provide trade-credit and financially constrained customers to rely on it, even if it comes at extremely high implicit interest rates. These topics have been analyzed in depth in Petersen and Rajan [1997]; Burkart and Ellingsen [2004]; Cuñat [2007]; Cuñat and García-Appendini [2012]; Giannetti et al. [2011]; Garcia-Appendini and Montoriol-Garriga [2013]. This strand of literature relies on credit rationing arguments and the observation that trade credit suppliers have a monitoring advantage over banks. On the contrary, in our model, the capital structure and the trade-credit joint financing decisions are simply determined by credit and liquidity risk-sharing

[2] The bargaining theory argues that debt improves a firm's bargaining position against its customers or suppliers (Bronars and Deere [1991]; Dasgupta and Sengupta [1993]; Hennessy [2009]; Chu [2012]). When a customer increases its leverage, it increases its bargaining power against its supplier. The supplier, unwilling to lose its bargaining power, may respond by increasing its own leverage. Therefore, the bargaining theory predicts a positive leverage relationship between the supplier and its customer. The relation-specific investment theory argues that debt discourages relation-specific investments made by both the supplier and the customer Jayant R. Kale and Husayn Shahrur [2007]; Hennessy [2009]; Chu [2012]). In that case, the supplier decreases its leverage when the customer increases leverage

motives with no reference to information frictions.

Additionally, the analysis of our dataset recovers and expands many stylized features documented in the extant empirical analysis of trade credit relations. In particular, Amberg et al. [2021b] show that trade credit positions are economically important sources of reserve of liquidity for firms. As trade credit adjusts through size variation and deferred payments, firms commonly increase trade credit borrowing during the contraction period of bank loans (see Nilsen [2002]) and delay repayments of trade debt during times of financial distress (Cuñat [2007]; Cuñat and García-Appendini [2012]) with the possible effect of increasing the business cycle fluctuation as in Jacobson and Von Schedvin [2015] that document and quantify the impact of trade credit on the propagation of corporate failure[3]. In particular, while financially sound firms can extend trade credit to their customers in the form of long payment terms, dampening the effect of a credit crunch on the economy, financially weaker suppliers might be exposed to liquidity shocks amplifying the effect of financial shocks.

Our paper complements these views, as it focuses on factoring, a (short-term) financial arrangement where the supplier sells the accounts receivables from trade credit at a discount and receives immediate cash from the factor, typically a bank or a financial service firm that can collect dedicated funds from capital markets, for immediate working capital needs. The existence of factoring helps financially constrained firms. As a result, our network modeling approach provides an important micro foundation to studies of the macroeconomic implications of the customer-supplier financial link and adds to the considerations of Grigoris et al. [2023], Ersahin et al. [2023], Luo [2020] and Bocola and Bornstein [2023]. In particular, the analysis of factoring relations in our sample highlights a strict connection between the mechanics of the supply chain of credit strategic equilibrium and the structural relations that underlie the characterization of granular volatility in firm networks given by Herskovic et al. [2020]. In our set-up, the supplier's asset evo-

[3]See also Klapper et al. [2012], Murfin and Njoroge [2015], Barrot [2016], Breza and Liberman [2017].

lution is explicitly affected by shocks that propagate along the customer-supplier links as identified by the factoring transactions. We augment these models by explicitly accounting for the role of a bank that, by offering a factoring service, finances trade credit offered by a supplier to the customer and provides independent support to the results of Giannetti and Saidi [2019] that prove that lenders have an active role in attenuating disruption of supply chains in distressed industries [4]. Conversely, in the event of a banking shock, the reduction in factoring and loans to suppliers could have a more pronounced impact on customers, potentially amplifying the output effects of a banking crisis.

The remainder of the paper is organized as follows. Section 2 describes our data and the empirical findings. Section 3 describes the model. In Section 4 we use a calibrated version of the model to match qualitatively and quantitatively some stylized facts about the interaction between firm capital structures, the use of factoring, and its pricing. Conclusions are in Section 5. The Appendix contains additional empirical analysis, the complete formal definition of the quantitative model, and the relevant proofs.

2 Empirical analysis

In this section, we provide an overview of the data that we use and present key statistics shedding light on the network structure shaped by factoring relationships. Our dataset is proprietary and originates from a major Italian bank group. It provides comprehensive insights into Italian firms that are clients of the bank, along with data on subsidiaries offering credit and factoring services. Each firm is uniquely identified by an individual ID, facilitating the seamless integration of various databases, as elaborated below.

The data, which covers the period from 2013 to 2015, is categorized into two distinct types: node information and edge information. Node information includes balance

[4] We do not consider explicitly the additional motivation for the use of factoring which is the decision of a firm to externalize the majority of the credit management functions (Mian and Smith Clifford W. [1992], Smith and Schnucker [1994]).

sheets, credit ratings, credit lines, outstanding loans, and credit registry data. On the other hand, edge information encompasses cash flows and factoring transactions. Access to the bank's factoring service requires suppliers to utilize it for discounting payments expected to be received from their customers. Further details, including extensive descriptive statistics of network-related firms identified through factoring transactions, are available in the Appendix.

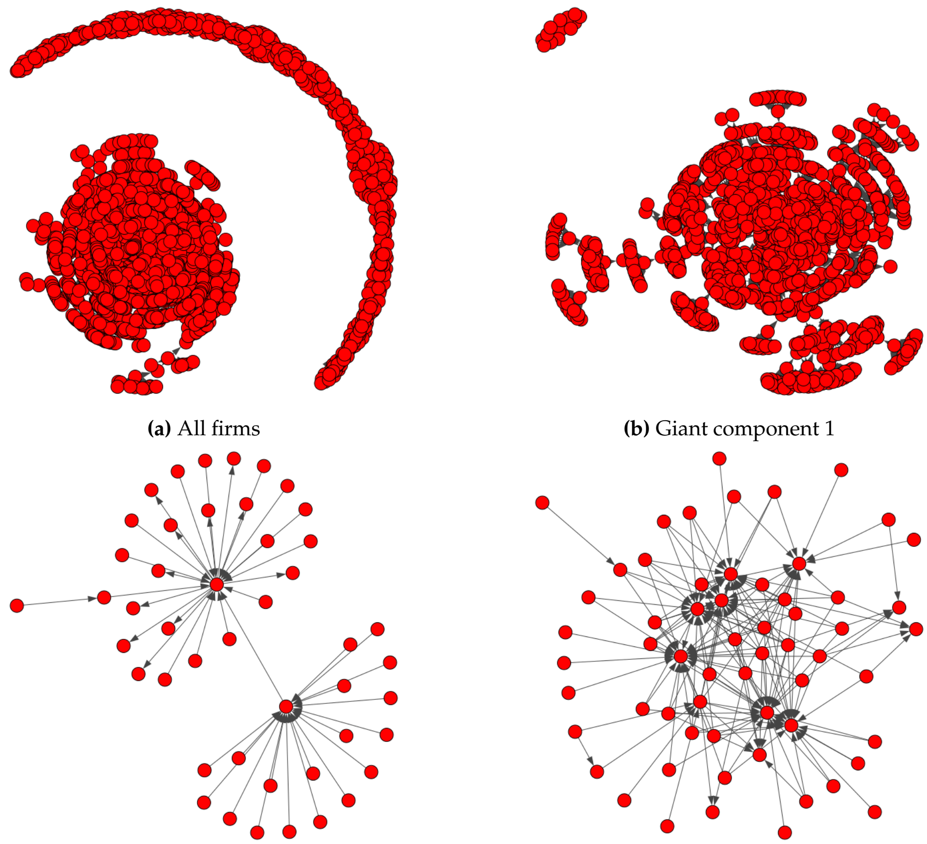
2.1 Factoring network

Using the *Infomap* algorithm of (Rosvall et al. [2009]), a widely employed method for detecting communities in directed networks, we identify the largest components within the trade credit network. Table (1) presents network properties for both the overall network and a few large clusters. The largest cluster comprises 48 percent of the network's firms, while the remaining components are smaller in size. Specifically, more than twenty firms belong to twenty clusters. In general, smaller clusters exhibit a lower average degree, which represents the number of connections each node has within the network, and a higher degree of dispersion, measured by the standard deviation of the degree. For instance, as shown in Table (1), the third component has a lower mean degree and a higher standard deviation compared to the giant component.

	Full Network	Giant Component 1	Component 2	Component 3
No. of firms	2663	1275	60	57
No.of suppliers	711	139	49	30
No of customers	1952	1136	11	27
Mean. degree	2.41	2.34	5.63	1.96
Min. degree	1	1	1	1
Max. degree	58	51	27	40
Std.dev. degree	4.37	4.34	6.14	5.55

Table 1: Network properties for trade credit firms in year 2013

Note that Infomap community detection makes use only of the edge information. The



(a) All firms

(b) Giant component 1

(c) Component 2

(d) Component 3

Figure 1: Firm network. Nodes represent individual firms and edges represent trade credit transaction.

summary statistics show that firms belonging to distinct components have different characteristics: Table (2) compares the key variables across the three components and the full firm network. Compared to the firms in the first two biggest components, the firms in the other components have on average lower liquidity, lower assets, lower sales growth, higher leverage, and more loan and credit utilization. They have comparable equity-to-asset ratios, operating costs, and fixed asset to total asset ratio. The network is fairly persistent over time. Figure (1) presents the network structure of the full and the first three components with the largest number of connected firms, over the three years of our sample.

A salient characteristic, evident through visual inspection and substantiated by quantitative analysis, is the tree-like topology of the network. In this topology, suppliers are linked to multiple customers, and circular relationships are minimal. For instance, let's

	Component 1	Component 2	Component 3	Remaining Components	All firms in Network
Liquidity*	950.27	1031.96	317.79	795.66	867.17
Total Assets*	28979.84	30325.94	9476.77	23927.98	26241.18
Sales Growth (%)	3.62	4.06	0.26	2.78	3.10
ROA	4.69	6.68	3.00	4.53	4.61
Leverage	6.03	4.64	6.92	7.23	6.57
MOL	1645.27	1837.45	546.06	1319.05	1473.54
Liquidity Ratio	0.04	0.04	0.04	0.04	0.04
Loan Utilization*	149.42	125.02	0.00	134.33	141.00
Loan Availability*	9940.91	6720.15	4455.58	7654.85	8754.12
Credit Utilization*	2025.18	558.47	3059.18	2490.93	2180.39
Credit Availability*	14067.11	7271.31	12400.00	13655.90	13549.34
Rating	6.38	8.25	7.79	7.19	6.80
Equity/Assets	0.28	0.30	0.44	0.29	0.29
Accounts Receivables/Assets	0.62	0.75	0.42	0.57	0.60
Operating Cost/Assets (%)	2.28	2.03	1.55	2.14	2.18
Fixed Assets/Assets	0.29	0.29	0.25	0.28	0.29

Table 2: Comparison of key variables across first three components and all firms for the year 2013. *in thousands of euros. All quantities refer to the average value.

examine the largest component: it encompasses 139 suppliers. To provide greater detail, among these 139 suppliers, 86 are categorized as 'root' suppliers, while the rest function as customers of these 'root' suppliers. Furthermore, within this component, 1,136 customers serve as end nodes, exclusively engaging as customers and not extending factoring services to other firms. These empirical observations indicate that star networks, with focal points around prominent suppliers, exhibit characteristics akin to "granular units."

In the following, we leverage on the additional information available in our dataset on firm balance-sheets and credit and factoring relations, to analyze the interaction between the factoring relationships and the capital structures of the firms.

2.2 Supplier's financial constraints and the factoring network.

In order to delve into the fundamental framework of the network established by customers and suppliers, and to investigate the influence of financial constraints on the factoring network, we analyze the repercussions of an external shock that eases the financial constraints of suppliers on their ability to access trade credit factoring.

Similar to Giannetti et al. [2021], we leverage the variations stemming from a reform introduced in 2014, which reduced the costs associated with purchasing and securitizing receivables for banks. This reform consisted of two legislative pieces. The first, known as the Decreto Legge 145/2013 (Decreto Destinazione Italia), approved in December 2013 and converted into law in February 2014, entailed the separation of securitized trade credit from the rest of the debtor's assets. The second, the Decreto Legge 91/2014 (Decreto Crescita), approved in June 2014 and converted into law in August of the same year, granted financial intermediaries increased flexibility in securitizing receivables. As a result, banks found it easier to supply factoring services. This development lowered the cost of offering trade credit for suppliers that could sell receivables to banks at reduced discounts, relying less on external financing to fund these assets. The impact of this reform is expected to be more pronounced for financially constrained suppliers.

Our analysis proceeds by estimating the consequences of the reform on two key aspects: the probability of providing trade credit factoring to new customers and its increased provision to customers that are already clients of the supplier. By alleviating the constraints faced by suppliers, we anticipate an increase in the use of factoring by the firms that are financially distressed, due to their high level of indebtedness. Consequently, using a linear probability model, we estimate the following relation:

$$\begin{aligned} New_t^{customer} &= b_0 + b_1 D_t^{rel} + b_2 reform + b_3 p_{t-1}^{default} \\ &\quad + b_4 reform * D_t^{rel} + b_5 reform * p_{t-1}^{default} \\ &\quad + b_6 p_{t-1}^{default} * D_t^{rel} \\ &\quad + b_7 reform * p_{t-1}^{default} * D_t^{rel} \\ &\quad + b_8 p_{t,cust}^{default} + \mu_j + \mu_i + \epsilon_t \end{aligned}$$

where the dependent variable, $New_t^{customer}(i)$ is a dummy variable equal to 1 if the supplier i extend trade credit to a new customer using factoring in period t. $D_t^{rel}(i)$ is a dummy that is equal to 1 if the size of the new customer[5] is bigger than the size of the supplier, measured at time $t-1$. This dummy can be interpreted, as in Giannetti et al. [2021], as a measure of the bargaining power of the customer; $reform$ is a dummy equal to 1 in 2015, since the reform has been approved and applied at the end of 2014; $p_{t,cust}^{default}$ and $p_{t-1}^{default}$ are the probability of default of the customer and of the supplier, respectively, and are a measure of financial distress estimated using the standard z-Altman score procedure relying upon balance sheet information ; μ_i and μ_j are sector fixed effects of the supplier i and the customer j, respectively. Standard errors are clustered at the supplier and customer sector levels. The coefficient b_7 of the triple interaction allows us to test if suppliers resort to factoring to expand trade credit to high bargaining new customers. Table 3, first column, shows that after the reform factoring services are used to extend trade credit to less financially distressed new customers, as the cost of factoring is lower for these firms. In particular, the negative sign on $p_{t-1}^{default}$ suggests that highly indebted suppliers are less likely to extend new trade-credit factoring lines to new firms before the reform. After the reform, arguably thanks to the decrease in the cost of factoring, there appears to be a higher probability for financially distressed suppliers to initiate new connections with larger customers. Transitioning from the lower percentile to the upper percentile of our primary indicator for supplier financial distress is linked with a rise of 18.61 percentage points in the likelihood of trade credit factoring towards high bargaining customers. Meanwhile, financially stable suppliers show a 9 percentage points increase in trade credit factoring extended to smaller customers, subsequent to the reform.

Then, we analyze how the reform affects the use of factoring for the provision of trade credit to customers that were already served by a supplier before the reform. We rely on

[5] We measure size with the log of asset

a similar regression specification:

$$\begin{aligned}\Delta Share_t^{exist.customer} &= c_0 + c_1 D_t^{rel} + c_2 reform + c_3 p_{t-1}^{default} \\ &+ c_4 reform * D_t^{rel} + c_5 reform * p_{t-1}^{default} \\ &+ c_6 p_{t-1}^{default} * D_t^{rel} \\ &+ c_7 reform * p_{t-1}^{default} * D_t^{rel} \\ &+ c_8 p_{t,cust}^{default} + \mu_i + \mu_j + \epsilon_t\end{aligned}$$

where $\Delta Share_t^{exist.customer}(i)$ is the change in the share of trade credit, financed by factoring, supplied to the already existing customers in period t. The second column of Table 3 shows that the reform results in a larger increase in the provision of trade credit factoring to relatively bigger and safer customers that are already in the portfolio of the supplier. Suppliers that face a higher cost of external finance before the reform, increase the supply of trade credit factoring to all their existing customers with no significant difference with respect to their size as the non-significant coefficient c_7 on the triple-interaction term shows. In general, existing safer customers that already enjoy trade credit factoring services after the reform enjoy a higher share of factoring provisions, as the cost of factoring is lower for these firms. The results are robust to the inclusion of customers and suppliers fixed effects.

The aforementioned discoveries validate that the redistribution of trade credit factoring, prompted by the external easing of financial constraints, yields varying effects across customers with differing bargaining power. Our findings supplement those presented in Giannetti et al. [2021]. With the reform alleviating suppliers' financial constraints, we can delineate the distinct responses of suppliers to the reform, contingent upon their level of financial distress.

In summary, our empirical results support the conjecture that the structure of the factoring network is the outcome of a complex bargaining procedure. The bank that provides

	(1) Factoring to a new customer	(2) Δ share of factoring to existing customer
rel.size	0.011	-.018
	(0.018)	(0.015)
rel.size*reform	-0.062***	.04*
	(0.020)	(0.021)
rel.size* $p_{t-1}^{default}$	-0.585**	.148
	(0.253)	(0.220)
rel.size* $p_{t-1}^{default}$*reform	1.318***	-0.202
	(0.222)	(0.279)
$p_{t,cust}^{default}$	- 0.057*	- 0.108**
	(0.032)	(0.042)
$p_{t-1}^{default}$	-0.018**	-0.322***
	(0.194)	(0.062)
$p_{t-1}^{default}$*reform	-0.754***	0.428***
	(0.243)	(0.068)
reform	0.088***	-0.057***
	(0.101)	(0.008)
sector fixed effect	yes	yes
overall R2	0.03	
year f.e.	yes	yes
N	11,892	10,375

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Effect of the 2014 Reform. Column (1) Probability of extending trade credit factoring to a new customer. Column (2) Increase in the trade credit factoring to existing customers

financial services, financially constrained suppliers, and customers accept to trade-off the benefits deriving from the adoption of the factoring service in support of the production relationship with the additional counterparty risk, that is the externality that is created by the new trade credit connections.

2.3 Factoring and firm capital structure.

We investigate now the relation between trade credit factoring and firms financial characteristics. The total amount of factoring in a given year is scaled by total assets and taken as a dependent variable.

	(1) Factoring / Supplier Total Assets	(2) Factoring / Supplier Total Assets
Collateral	0.0078	0.0157
	(0.0234)	(0.0262)
Size	-0.0187***	-0.0170***
	(0.0028)	(0.0027)
Operating Costs/Assets	0.0504***	0.0482***
	(0.0058)	(0.0062)
Equity/Asset	0.0289***	0.0140
	(0.0094)	(0.0104)
Account Receivables/Assets	0.0483*	0.0525**
	(0.0192)	(0.0198)
Account Payables/Assets	0.2058***	0.2093***
	(0.0427)	(0.0388)
D^{rating}		-0.0235***
		(0.0066)
Cons	0.1354***	0.1863**
	(0.0379)	(0.0838)
year fixed effects	yes	yes
sector fixed effects	yes	yes
N	1703	1609
R^2	0.352	0.339

p-values in parentheses
$^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 4: Factoring amount against suppliers characteristics at time t-1.

In our model, factoring is used as a financing source by the firms. Therefore, we test if there exists any association between the use of factoring and the availability of credit to firms. We do find evidence that firms use factoring as a source of financing of working capital and as an instrument of cash-flow improvement.

We perform our analysis starting from the suppliers and then we move to the customer level. The dependent variable used throughout this section is *trade credit factoring divided by total assets*. As independent variables of interest, we use *collateral* constructed as the total amount of fixed assets over total assets, *equity over assets*, calculated by subtracting

	(1) Factoring / Customer Total Assets	(2) Factoring / Customer Total Assets
Collateral	-0.0092***	-0.0106***
	(0.0018)	(0.0021)
Size	-0.0036***	-0.0039***
	(0.0003)	(0.0003)
Operating Costs	0.0081***	0.0085***
	(0.0009)	(0.0009)
Equity/Asset	-0.0004	0.0006
	(0.0006)	(0.005)
Account Receivables/Assets	-0.0176***	-0.0197***
	(0.0024)	(0.0027)
Account Payables/Assets	0.0297***	0.0286***
	(0.0047)	(0.0048)
D^{rating}		0.0009*
		(0.0005)
Cons	0.0327***	0.0356***
	(0.0038)	(0.042)
year fixed effects	yes	yes
sector fixed effects	yes	yes
N	19586	18123
R^2	0.065	0.065

p-values in parentheses
$^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 5: trade credit factoringamount received by customer against customers characteristics.at t-1

total liabilities from total assets, *size* measured by the log of assets, *operating costs*, that include direct costs of goods sold and other operating expenses[6], as a control for the level of activity of the firm, *account receivable divided by assets* and *account payable divided by assets* which measure trade credit supplied and received over assets.

The first column of Table(4) shows how the use of factoring is higher in firms with high level of equity over assets, that is with low leverage. The effect is statistically significant.

[6]Other operating expenses include rent, payroll, and other overhead costs, as well as raw materials and maintenance expenses

In economic terms, going from the first to the third quartile of the distribution of the equity/asset variable translates into an expected increase in factoring of 0.92 percent which is quite relevant considering that the median value of factoring divided by total assets is 7.7 percent.

The results show that smaller size firms use proportionally more factoring. Additionally, they suggest that suppliers use factoring to meet liquidity needs and highlight the importance of factoring as a source of insurance by the suppliers and of short-term financing, which has not received sufficient attention in the literature.

The insurance mechanism is further supported by a positive coefficient on the operating costs and on payable accounts. Companies are looking for cash to pay operating expenses and pressing financial obligations. Hence, they choose to factor to extend their customer's payment terms. By not providing extended payment terms, a contract sale or a customer relationship can be lost. Another possible explanation for the positive coefficient on payable accounts is that companies choose to focus on their business day-to-day operations and outsource their accounts receivable department. A good factoring company, like a bank, can help firms make significant reductions in losses due to non-payment by assisting in analyzing the credit of firm customers before entering into a contract with them and delivering goods. Hence, the higher the account receivables the higher the need for factoring.

Firms that are credit constrained have more difficulty accessing factoring services. The second column of Table (4) reinforces this hypothesis. We control for the rating as a proxy for credit constraints. In particular, we control for the dummy D^{rating} which is equal to one when the firm is in distress[7].

Next, in Table (5), we analyze the relationship between trade credit factoring and customer characteristics. Similar to the case of the suppliers, smaller customers with high

[7]Rating measures the credit score of a firm, whose value ranges between 2 and 14. The higher the value, the lower the credit valuation of the firm. The dummy rating, D^{rating}, is equal to 1 if the rating is greater than 7 which is the usual threshold for a firm to be considered in distress.

operating costs, use more trade credit factoring especially when , in the presence of obligations to pay off a short-term debt to its creditors or suppliers, accounts payable become due faster than the terms of payment under the accounts receivable. This effect is also evident for customers that are more credit constrained and have a lower level of liquidity, as Column 2 of Table(5) shows when controlling for the rating of the customer. As expected, credit constrained cusomer use more trade credit factoring.

3 The Model

We formulate the dynamic of supplier assets in continuous time. For simplicity, we assume that bank's and the firm's strategic decisions and payments take place in two periods of fixed length T. At the time $t = 0$ a customer c borrows from a bank and the loan is to be repaid at time $t = T$. The customer also borrows in the form of trade credit from a supplier. If the customer is unable to repay both the bank as well as the supplier, then she becomes insolvent and her assets are liquidated. We assume that the bank loan repayment is senior in case of a default. At $t = T$, if the customer is able to repay the bank, but not the supplier, she can exercise an option to delay the repayment of the amount due to the supplier to time $t = 2T$, to avoid insolvency. The customer hopes to receive a positive cash flow shock between $t = T$ and $t = 2T$ to be able to repay in full the amount due to the supplier and avoid bankruptcy. Let us consider the individual positions of a representative customer, of a supplier, and of the bank.

3.1 The customer problem

Customer assets are driven by lognormal dynamics and the equilibrium capital structure is determined by a standard trade-off between the tax benefit of debt (τ tax rate) and costly default (α bankruptcy cost). At time 0 the customer issues a zero coupon bank debt security valued at V_D^c with a promised repayment of R_D^c, and issues a trade credit security to the supplier valued at $V^c_{\mathcal{F}}$ with a promised repayment of $R^c_{\mathcal{F}}$. The firm becomes insolvent at $t = T$ only if it is unable to repay the bank. In fact, the customer has the option at time $t = T$ to delay the repayment of the trade credit to time $t = 2T$. This option represents the additional insurance value offered by the trade credit contract. Hence the value of trade credit embeds the option value of postponing the payment to $t = 2T$. Both the default thresholds at time T and at time $2T$ are determined endogenously. The customer equity value is easily determined since the equity holder is the residual claimant of the expected cash flows once all debt has been repaid.

3.2 The supplier problem

A supplier s is connected by a star network to a number N^s of customers through trade credit flows financed by the factoring service. She raises equity in the competitive capital market to finance her operations, raises debt from the bank, and offers a trade credit facility to customers. In particular, the supplier raises a loan with a face value of V^s_D and promised repayment of R^s_D. The amount of trade credit is in the form of accounts receivables, which means that the customer promises to pay the supplier the amount $R^c_{\mathcal{F}}$ in the future for the borrowed trade credit amount priced at $V^c_{\mathcal{F}}$. Similar to the customer, the capital structure of the supplier is set endogenously according to the trade-off between the tax benefit and the bankruptcy cost, where the default threshold value is determined endogenously.

The trade-credit position extended to each customer j is priced at $V^c_{j,\mathcal{F}}$ with a promised repayment amount of $R^c_{j,\mathcal{F}}$. The total value of the trade credit to the supplier is simply the sum of the trade credit values issued to her customers. That is,

$$V^s_{\mathcal{F}} = \sum_{j=1}^{N^s} V^c_{j,\mathcal{F}} \tag{1}$$

where $V^c_{j,\mathcal{F}}$ denotes the value of trade credit issued by the supplier s to the individual customer j. The propagation of asset risk along the supply chain is captured in the equation describing the dynamics of the supplier assets:

$$\frac{dA_t^s}{A_t^s} = \gamma \sum_{j=1}^{N_c} w_j \tilde{\Delta}_{j,t}^c \frac{dA_{j,t}^c}{A_{j,t}^c} + \sigma_s dZ_t^s - l dH_t \tag{2}$$

where Z^s is s standard Brownian shock that induces a volatility, H_t is a binary random variable that takes value 1 with probability λ_s and 0 otherwise, and l is a constant jump size. The variable H_t captures the liquidity risk. The first component, on the r.h.s. represents the growth opportunities emerging from the trade credit relationship. The variable $\tilde{\Delta}_j^c = \frac{A_{j,c}}{\Pi_j^c} \frac{d\Pi_j^c}{dA_j^c}$ is the elasticity of the composite option value with respect to the underlying, where Π_j^c denotes the value of the composite trade credit position, including the optional value. Details about its computation are given in Proposition (7) in the Appendix. An adverse liquidity event ($\lambda_s > 0$) reduces profitability (the growth rate of the assets). When $H_t = 1$, the value of assets gets reduced by a proportion l since $l < 1$.

Note that equation (2) introduces the key network interaction, assets of suppliers are exposed to customer's asset shocks weighted by the proportion of trade credit amount transacted between the two parties, and parameter γ captures the strength of the network propagation effect. The interpretation of γ is similar to Herskovic et al. [2020]: a value close to zero indicates no network effect, whereas a value that is close to 1 indicates a strong network effect. The presence of customer trade credit increases the volatility of supplier assets by an amount $\gamma^2 \sum_{j=1}^{N^s} \left(\tilde{\Delta}_j^c w_j \sigma_j^2 \right)^2 = \gamma^2 H_{out}$ where H_{out} is the Delta-volatility-adjusted Herfindahl-out concentration index for the supplier s . The individual weight w_j will depend on the industry-specific relationship between the customer and the supplier, while the delta contribution accounts for the impact of the customer's financial riskiness. Note that the extent of diversification depends on the concentration of trade credit positions since the shocks that propagate through the network channels cannot be fully diversified. Therefore, the model embodies granular volatility effects as in

Herskovic et al. [2020]. That is, the volatility of the supplier's (and indirectly of bank) assets is increasing in the concentration that is measured by a delta-volatility-adjusted Herfindhal-out index.

3.2.1 Financially constrained supplier

We introduce an exogenous financial constraint on the supplier's debt to cope with its exogenous relaxation after the regulatory change discussed in section 3, Table (3). We follow Kiyotaki and Moore (1997) and impose a constraint that prevents the supplier from borrowing more than a fraction of total asset value.

$$V_D^s \leq \kappa(V_D^s + V_{\mathcal{F}}^s + V_E^s) \tag{3}$$

Thus, the relaxation of the constraint will be mapped to an increase in the level of the maximum allowed leverage parameter κ.

3.2.2 The interaction between financial and production decisions.

Equation (2) relates the risks influencing customer asset dynamics and the dynamics of a supplier's assets. This effect is modulated by a weight, denoted as w_j, which is multiplied by a metric of customer risk exposure, denoted as $\tilde{\Delta}_j^c$. This metric takes into account individual customer leverage and counterparty risk. The weight w_j serves to quantify the relative significance of a customer within the portfolio of customers served by the supplier's production. Production decisions and trade credit allocations of firms have been the subject of a comprehensive study in the literature. In particular, Giannetti et al. [2021] provide evidence that suppliers employ a sales strategy with quantities and trade credit contract terms. This strategy is designed to alleviate competition in the downstream market. Our model speaks to this literature through the endogenous formation of weights w_j. This decision of how much trade credit to issue not only factors in the financial risk profiles of the relevant stakeholders across the supply chain, but also accounts for the production capacity of customers competing in the downstream market, as explained next.

To quantitatively assess the trade-offs encountered by customers and suppliers in navigating financial and production decisions, we introduce a model extension. In this extension, both the bank and the supplier cater to two distinct customers, denoted as $j = H, L$, each possessing varying levels of bargaining power. These customers engage in a Cournot competition in the downstream market, and choose the optimal trade credit, leading to an endogenous w_j. Similar to Giannetti et al. [2021], the supplier offers a state-contingent contract to the customers that depends on the realization of a demand shock in the production process.

We consider a determination of optimal weights under two counterfactual scenarios. As in Giannetti et al. [2021], we consider first the supply of trade credit to the H-customer for an unlimited amount of input goods sold, at an implicit interest rate that is below its cost of capital by a dollar amount ϕ. This sub-optimal policy implies a cannibalization of sales to the L-customer when the realized demand shock is not large enough. In order to minimize the ex-ante risk of cannibalization, trade credit is designed to have the features of a credit line conditional on input purchases. In particular, the supplier optimally chooses a credit limit up to a determined dollar value of goods sold smaller or equal to a dollar amount x. In this second scenario, the supplier can maximize her revenues by simultaneously accommodating the participation of the H-customer while minimizing the ex-ante probability that the L-customer is driven out of the downstream market. This feature of the contract allows the supplier to target inframarginal units and therefore leaves unaffected the customer's marginal cost and consequently the downstream market price.

3.3 The bank problem

The bank issues loans to both the suppliers and the customers who use their respective cash flows A_T^s and A_T^c as collateral with promised repayment of R_D^s and R_D^c. The bank receives the promised repayment as long as the collateral is above the respective thresholds ζ^c and ζ^s. In case of default, the collateral is taken over by the bank, after the bankruptcy cost α_b. The bank borrows a nominal amount of B from competitive debt and equity markets to issue loans to the firms. The promised repayment on the zero coupon bond that the bank borrows is R^b, and the interest paid on the debt is tax deductible. The bank defaults whenever the payoff on loans is lower than the promised repayment amount. Since the banks operate in a competitive market, the spread δ^b is determined such that the bank makes zero profit in expectation.

Note that, from an institutional perspective, the bank is obliged to implement a risk mitigation strategy. This strategy involves selecting a lending policy that minimizes exposure to firm-specific shocks. Consequently, the bank takes measures to minimize its loan portfolio's risk through diversification of firm-specific shocks. However, a key distinction from Gornall and Strebulaev [2018] is that, due to the factoring services, the aggregate bank payoff and riskiness depend on the structure of the network of trade credit relationships. This is because any shock affecting the customers directly impacts the assets of the suppliers, which in turn affects the bank's assets. As a consequence, the optimization problem of the bank is considerably more intricate due to the presence of a complex web of interactions and correlations among firm cash flows.

3.4 Equilibrium

We formulate the joint optimization problem to determines the equilibrium capital structure and trade credit relationship for a single cluster formed by the bank, the supplier and N^s customers. The vector with N^s components of face values and nominal reimbursed amount(s) of the factoring service will be univocally denoted $V_{\mathcal{F}} = \left(V_{j,\mathcal{F}}\right)_{j=1,..,N^S}$ and $R_{\mathcal{F}} = \left(R_{j,\mathcal{F}}\right)_{j=1,..,N^S}$. Then optimization will iteratively determine the vector of notional debt repayments $R := \left(R^b_D, R^s_D, R^c_D, R^c_{j,\mathcal{F}}\right)$.

Optimizing for the general configuration of a star network where each supplier has an arbitrary number N^s of customers, is out of the scope of this paper.[8] To gain more intuition on the interaction between supply-chain financial relations and firm capital structures we restrict the analysis to two benchmark cases: i) the simplest configuration where we can set $N^s = 1$, i.e. the bank offers lending and factoring services to a supplier and to an individual representative customer and ii) the case $N^s = 2$ with two customers having differential bargaining power, competing in the downstream market and served by a financially constrained supplier. Then we state the following:

Theorem 1 *Assume a competitive banking sector that offers, in addition to a standard bank-credit line, also a second separate credit line dedicated exclusively to factoring services that suppliers may offer to their customers. The capital for these two credit lines is collected directly from the external market through separate debt issuance. Then the joint capital structure of the bank and the firms is determined by the maximization of the functional:*

$$V(R) = V_D^b + \sum_{j=1}^{N^S} V_{j,\mathcal{F}} + V_E^b + V_E^s + V_E^c$$

with respect to the vector of notional debt repayments $R := \left(R_D^b, R_D^s, R_D^c, R_{\mathcal{F}}\right)$, *and the optimal solution* R^* *must verify*

$$V_E^{b*} + V_D^{b*} = V_D^{s*} + V_D^{c*}$$

Proof. As a straightforward extension of the solution from Gornall and Strebulaev [2018], we consider a conglomerate financial institution formed by the bank+supplier and we assume it offers, in addition to bank loans, also the factoring service supporting the supplier's marketing of the produced good. Results follow immediately from assuming that the bank and the firms jointly maximize the total value of the financial conglomerate. ■

[8] In fact, the optimization would suffer a curse-of-dimensionality problem. In addition, the resulting solution would be strongly dependent on the network weights which are assumed in our framework to be exogenous. However, as the empirical results highlight, these weights are an endogenous outcome of the interaction between productive and financial relationships, an additional layer of complexity to the problem. The general case will be relevant to understand the dynamic readjustment of the allocation of credit and factoring services within the bank portfolio which is not the main focus of this paper and is left for future research.

Remark 2 *Note that:*

i) The insolvency thresholds and the probability densities are unaffected by a homogeneous transformation of the debt repayment vector $\left(R_D^b, R_D^s, R_D^c, R_{\mathcal{F}}^c\right)$.

ii) All equity and credit (state contingent) payoffs are linear (homogeneous of order 1) functions of the debt repayment vector $\left(R_D^b, R_D^s, R_D^c, R_{\mathcal{F}}^c\right)$.

Customers and the supplier adopt a shareholder equity maximization principle constrained by the condition that their debt, which includes also the trade credit, must be funded. The notional amount of trade credit $R_{\mathcal{F}}^c$ is selected considering among the admissible configurations of $R := \left(R_D^b, R_D^s, R_D^c, R_{\mathcal{F}}^c\right)$, the one that maximizes the total value of the conglomerate formed by the bank+supplier+customer.

3.5 Pricing of Bank credit and of Factoring service

Beyond optimal allocations, the resulting equilibrium provides structural information about the prices of bank loans and of trade credit factoring relating them to the relevant stakeholder characteristics and of the structure of the lending network.

In order to quantify the impact of different sources of risk and different configurations of the network, we define a number of interest rate spreads. First, following Gornall and Strebulaev [2018], we introduce a bank credit spread δ^b that determines the cost associated with the bank portfolio credit risk. Note that, in our extension, this spread will depend not only on the riskiness of the pool of customers but also on the overall configuration of the network of trade and bank lending relationships. Individual customer spreads δ^c for bank credit are implicitly determined by the single customer or supplier pairs (V_D^c, R_D^c).

On top of the credit spreads, we can quantify the competitive price that is charged for the liquidity service that is offered by the factoring trade credit, as measured by an upfront spread, δ^{tc}. All else equal, the unit notional price of trade credit factoring will be higher than the bank credit for the customers, since it offers an additional service in the form of an option to delay the payment to the supplier. Note also that, in light of the purely competitive assumption in the banking industry and of the risk-neutral pricing approach, this price does not account for any markup determined by differential information and it is simply dependent on the expected cash flows, assuming the existence of a liquid market for credit risk. From this point of view, this price is the first best outcome that takes into consideration only the full information-sharing risk-return trade-off [9]. In this respect, we complement the consolidated approach to trade-credit valuation that has been proposed in Burkart and Ellingsen [2004] taking into consideration the differential ability of stakeholders and investors to observe the cash flows and monitor credit repayments.

3.6 Numerical determination of the equilibrium values

The quantities $\boldsymbol{x}^* = [R^c_{\mathcal{F}}, R^c_D, R^s_D, R^b, \delta^{tc}, \delta^b]^T$ are determined in equilibrium by fixing a set of initial values $\boldsymbol{x}^{(0)}$ and then considering a sequence of numerical optimization steps utilizing the 'fmincon' routine in MATLAB. At each step the optimization function is defined in terms of the model equations as follows:

- For each iteration in the numerical solver, given the set of optimal decision variables $\boldsymbol{x}^{(n)}$, time 0 prices V^c_D, $V^c_{\mathcal{F}}$, V^c_E, V^s_D, $V^s_{\mathcal{F}}$, V^s_E, V^b_D, V^b_E and the threshold parameters ζ^c,$\hat{\zeta}^c$,$\bar{\zeta}^c$,ζ^s are jointly determined from the system of equations (19, 10, 16,28) and equations (20, 29, 17,30, 34 and 35). Solving the system of equations entails a fixed-point numerical scheme. The expectations are computed by making use of a hybrid analytical and Monte Carlo computation method.

- In each iteration the value of the spreads is determined using the relations

$$\delta^{b(n)} = \underset{\delta^b}{argsolve} \quad V^b_D + V^b_E = V^s_D + V^c_D \tag{4}$$

$$\delta^{s(n)} = \underset{\delta^s}{argsolve} \quad V^s_{\mathcal{F}} = \tilde{V}^s_{\mathcal{F}} \tag{5}$$

[9] Within this approach, the pricing of pro-soluto and pro-solvendo factoring schemes is simplified: they differ only in relation to the payer of the additional default risk, under the assumption that counterparties are equally and perfectly informed.

where $\tilde{V}^s_{\mathcal{F}}$ is determined by equation (32).

The procedure is repeated until the numerical scheme optimizes equation (4) with an accuracy of $1e-15$. The prices of all the relevant claims is determined by the resulting set of decision parameters $[R^c_{\mathcal{F}}, R^c_D, R^s_D, R^b, \delta^{tc}, \delta^b]^T = \boldsymbol{x}^*$.

4 Model based analysis

We study the variation of the optimal allocation of trade and bank credit as a function of the exogenous parameters of our structural model in order to match the empirical evidence in section 2. As previously explained, we focus our analysis on the cluster formed by the bank, a supplier, and a single customer. The supplier can lend to the customer by extending a trade-credit line supported by the bank factoring service.

Fundamental parameters shown in Table (6) are set to fixed benchmark values. Our conclusions are robust to a broad range of variations in these parameters. The tax rate is taken to be 25% which is close to the Italian Corporate Tax rate. Gornall and Strebulaev [2018] also use a value of 25%. We take the bankruptcy costs to be 10%, in line with James [1991] and Bennett et al. [2015]. The network propagation parameter γ, which governs the strength of the network, is set to 0.99. While this value is higher than the value of 0.9 set in Herskovic et al. [2020], we verified that in our optimization there is little variation ($\leq$ 3%) in the interval between 0.85 and 1 and we set a value 0.99 close to 1 to capture the limiting case where the seemingly idiosyncratic shocks propagated by the trade-credit relation have the strongest impact. Lastly, the correlation of shocks in the bank portfolio is taken to be 30%. This value is consistent with Basel I and Basel II regulatory requirements and is not far from the value of 20% considered in Gornall and Strebulaev [2018].

The cluster formed by the bank, the supplier, and the customers, is exposed to a Bernoulli liquidity shock and to three diffusive shocks affecting the overall configuration of the network: the one driving the customer's assets, the one driving supplier's assets

Parameters	Value
Bankruptcy cost (α)	0.1
Tax rate (τ)	0.25
Network propagation (γ)	1.0
Correlation of shocks (ρ)	0.3

Table 6: Calibration values

and, finally, a common systematic shock.

Our calibrated structural model rationalizes a number of empirical findings in addition to the ones related to the supply chain of finance in Gornall and Strebulaev [2018]. The additional findings of our calibrated model come from the fact that the firms are separated into suppliers and customers, who are connected by a trade credit network. In Gornall and Strebulaev [2018], the seniority of bank debt combined with the loan diversification benefits explains high bank leverage compared to firm leverage. In our model, firm debt is differentiated between trade credit factoring and bank credit and its composition is also relevant. Hence, to avoid ambiguities, firm leverage will be parametrized by the ratio between equity capital and total firm assets with the convention that a high (low) leverage corresponds to a low (high) equity-to-asset ratio. In the Appendix, we perform a sensitivity analysis with respect to exogenous parameters that documents the ability of our model to provide information sufficient to run a strategic equilibrium analysis of the two competing forms of debt: bank credit and a factoring service, with this last one offering customers additional insurance with respect to liquidity shocks.

4.1 Capital structure determinants of trade credit

In this subsection, we explore the relation between capital structure decisions of firms and the amount of trade credit financed with factoring offered by the supplier to the customer,l. We investigate the ability of the model to reproduce the relationship between customer and supplier capital structure and the intensity of the factoring relationships documented in Tables 5 and 4.

First, for a fixed benchmark level of the systematic shock common to all firms, we compute the range of equilibria spanned by independently varying the intensities of the three firm-specific exogenous shocks. We vary σ^s, the volatility of supplier shocks, σ^c, the volatility of customer shocks, and l, the jump intensity of the liquidity shock and analyze the variation of the equilibrium capital structure and of the factoring service. Equilibria are computed sampling for each parameter σ^c, σ^s, l uniformly the interval $10\% - 90\%$.

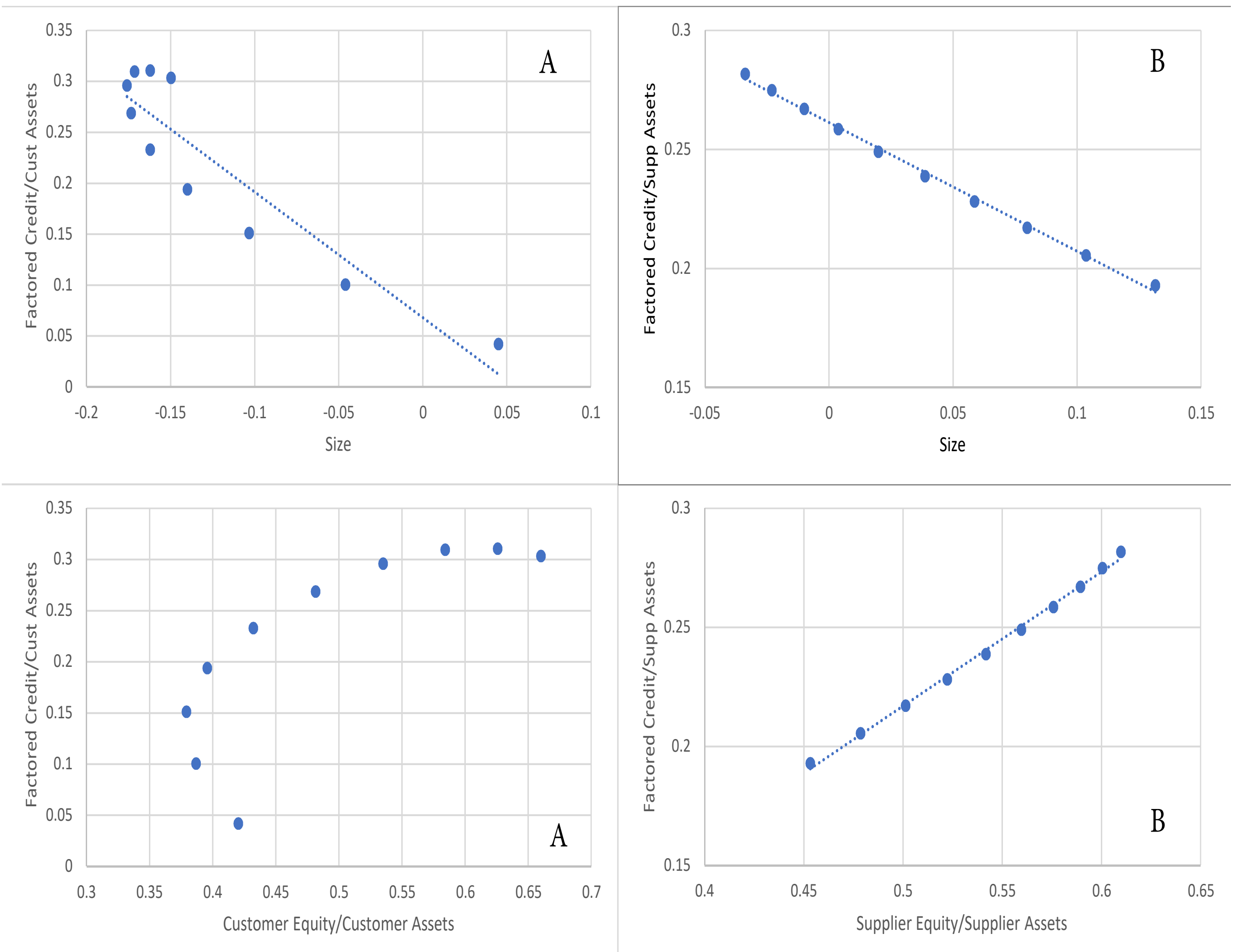


Figure 2: Cross-sectional relationships between trade credit factoring/assets vs size and vs equity capital for customers (inset A) and suppliers (inset B). Dashed lines correspond to linear interpolation lines.

Fig.s 2 shows, respectively, the variation of the fraction of trade credit factoring (Fac-

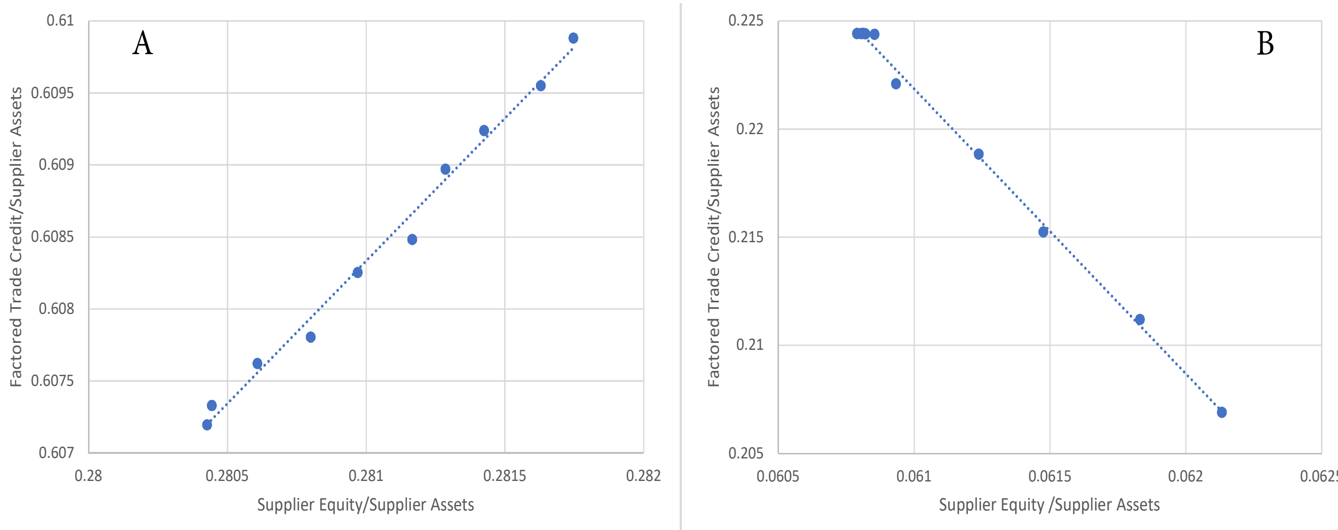


Figure 3: Marginal variation of trade credit factoring for a small variation of the equity capital for a low-leverage firm non-credit-constrained firm (Inset A) and a high-leverage, credit-constrained firm (Inset B)

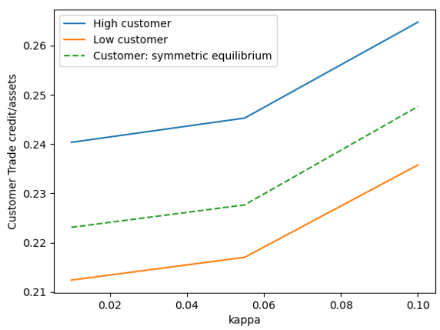
tored Credit/Assets) as a function of firm size (the log of firm Assets) and the measure of risk capital (Equity/Assets). Inset A reproduces the findings for customers while Inset B displays the same quantities for suppliers. In this numerical test the customer and the supplier asset volatilities are kept constant, while the intensity of the liquidity shock is varied. Note that the exogenous variation of the intensity of the liquidity shock reproduces the joint variations, detected in the cross-sectional regressions, for both suppliers and customers. That is, for suppliers, the fraction of trade credit financed with factoring is increasing in equity or equivalently decreasing in leverage. For both suppliers and customers, a higher asset size implies a lower recourse to factoring. Indeed a higher size implies higher resilience to exogenous liquidity shocks and a lower necessity to demand the liquidity protection provided by the factoring service. Customer leverage is not a significant determinant of recourse to factoring in the regression and correspondingly, in the chart, the relationship between the measure of trade credit factoring and that of leverage is non monotonic.

With the exception of the relation between the fraction of trade credit factoring and the level of equity capitalization of the supplier, the above conclusions are robust and remain virtually unaffected by changes of the customer asset volatility (data are available upon request). Figure 3 is obtained by sorting the numerical simulation output with respect to the level of suppliers' capitalization and considering two extreme situations: Inset A considers a situation where suppliers that are well-capitalized offer trade credit to low-risk customers, while inset B considers the case where suppliers that are highly levered offer trade credit to high-risk customers.

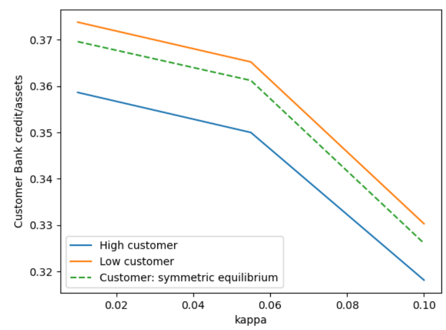
As shown in inset A, as the credit constraints are eased, a marginal increase in supplier equity capital drives a marginal increase in the amount of trade credit factoring. On the contrary, Fig. 3, inset B, shows that a marginal increase of supplier equity capital drives a marginal decrease in trade credit factoring for highly levered and credit-constrained suppliers. In other terms, suppliers who do not face credit constraints are inclined to maximize the advantages offered by the factoring service. Conversely, at the margin, poorly capitalized and credit constrained suppliers reduce the size of their trade credit factoring position to reduce the exposure to liquidity shocks that might propagate through the trade credit network. This result matches and is supported by the empirical result documented in the second column of the table 4 : the credit constraint dummy is negative and significant and renders insignificant the (positive) regression coefficient of the supplier equity-to-asset ratio. Thus, the above model and empirical findings indicate that the level of the supplier equity capital works like a buffer of risk capital whose level, jointly with the size of the trade-credit services offered, is determined by the severity of the credit constraints and by the necessity to reduce the impact of shocks on operations that may be transferred through the trade-credit relationship.

4.2 Extended model: Financially Constrained Supplier with customers competing in the downstream market

In this subsection, we illustrate the main quantitative findings from an extended model where a financially constrained supplier serves two-customers competing in the downstream market. The two customers differ as the H-customer, has a higher bargaining power when contracting with the supplier . We assume a uniform distribution for the demand shock, $\alpha \sim U[\underline{\alpha}, \bar{\alpha}]$, and benchmark parameters $\phi = 0.3, c = 0.3, \underline{\alpha} = 1, \bar{\alpha} = 2, K = 0.15$ to calibrate the model of downstream competition. The supplier asset volatility is set equal to parameters set equal to: $\sigma^{H,c} = \sigma^{L,c} = 0.4$.



(a) Customers's trade credit factoring against fin. constraints of supplier under a no-cannibalization policy equilibrium and a symmetric equilibrium.



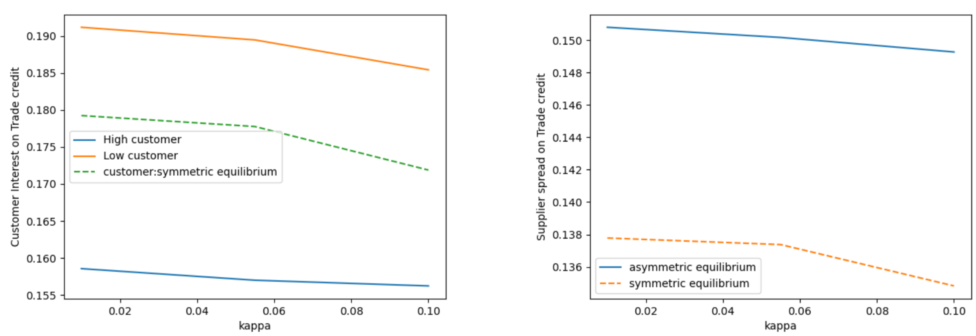
(b) Customers' bank credit against fin. constraints of supplier under a no-cannibalization policy equilibrium and a symmetric equilibrium.

Figure 4: Impact of financing constraints on trade credit factoring and bank credit. Larger kappa represents relaxation of suppliers' financial constraints.

First, we compare the optimal policy, the one avoiding cannibalization of sales, with the symmetric equilibrium corresponding to the situation where no discount on the cost of capital is applied.

Figure (4) illustrates the increase in the share of factored trade (inset A) and a reduction in bank (inset B) credit as the financial constraint is relaxed. The amounts (normalized by assets) received by the H-customer and the L-customer compared to the symmetric equilibrium case show that differential bargaining power drives a wedge, with the quantity provided to the H-customer being higher. The above findings confirm that the reallocation of factoring services, due to the exogenous relaxation of financial constraints, has an unequal impact among customers with different bargaining power, favoring H-customers. This is consistent and rationalizes the empirical evidence in Table 3 that after the 2014 reform, the relaxation of supplier constraints increased the trade credit factoring provision to the customers that are larger and to the one that have lower counterparty risk. The variation of the shares as the financial constraint is eased is not regular. The kink signals the shift from an equilibrium where the financial constraint is binding to a situation where there is slack. Note that, at the right of the kink, the substitution of bank-credit with factored trade-credit takes place at a faster pace.

Figure (5) inset A shows the variation in the price of the trade credit factoring position as the financing constraints are relaxed and a large demand shock is realized. It is interesting to observe that the discount on the cost of capital offered to the H-customer w.r.t the one offered to the L-customer is slightly reduced as the credit conditions are eased. Moreover the overall spread of the supplier bank credit (inset B) is decreasing with easing credit conditions and is reduced by the presence of downstream competition. Note that this equilibrium model output provides a structural explanation to the observation in Herskovic et al. [2020]: a larger dispersion in sizes (here proxied by differential bargaining power) raises supplier asset risk.



(a) Customers's trade credit interest rate against fin. constraints of supplier under a no-cannibalization policy equilibrium and a symmetric equilibrium..

(b) Supplier spread against fin. constraints of the supplier under a no-cannibalization policy equilibrium and a symmetric equilibrium..

Figure 5: Impact of financing constraints on trade credit value. Larger kappa represents a relaxation of suppliers' financial constraints.

As a counterfactual, to assess the financial value of the no-cannibalization of sales policy, we compute the ex-ante probability that L-customer does not participate in the downstream market for two sale policies feasible by the customer: (a) the optimal, no-cannibalization of sales, policy and (b) the cannibalization policy for small and large demand shocks. As expected, the probability that the L-customer is driven out of the downstream market, due to the aggressive behavior of the H-customer, is higher under the sec-

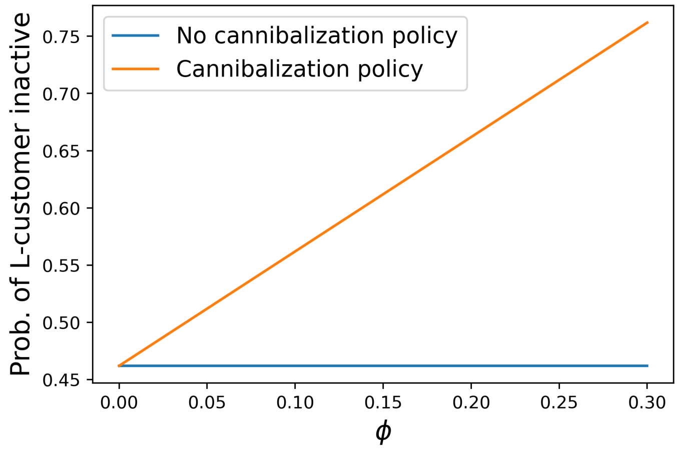


Figure 6: Probability of cannibalization under policy (a) optimal, no cannibalization of sales (blue line), and policy (b) the cannibalization policy for small and large demand shocks (red line).

ond policy with the probability increasing in the discount ϕ provided to the H-customer under the cannibalization policy, as shown in Figure (6). With increasing discount ϕ the probability remains constant under policy (a) while it is linearly increasing under (b).

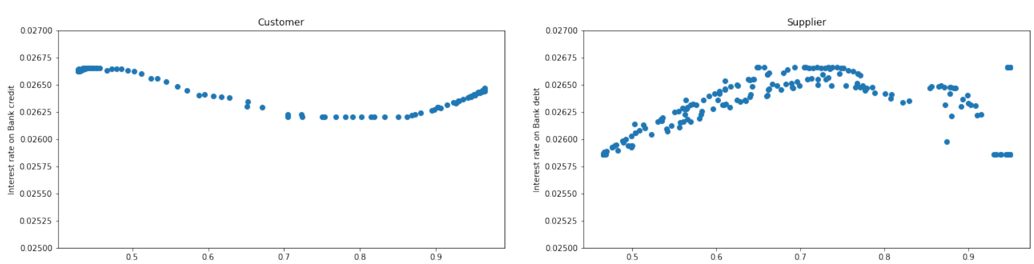
This model assessment provides a quantification of the interaction between the productive choice of the supplier, that is the two sale policies (a) or (b) and the supplier asset risk. We computed the ex-ante 'granular' volatility taking into account the network effects that, in turn, are endogenously determined by the adopted sale policy. Note that, the variance of the supplier's asset risk is related to the trade credit quantities by:

$$SupRisk = \gamma^2 \left(\sum_{i \in \{H,L\}} \left(\tilde{\Delta}^{i,c} \frac{q^i}{Q} \sigma^{i,c} \right)^2 \right) = \gamma^2 H_{out} \tag{6}$$

where $Q = q^H + q^L$ is the total quantity of trade credit demanded by the customers. In the presence of granular customers, such as those whose volatility is large and/or whose relative quantity demanded is large, the supplier's credit risk is higher. Since ex-ante the two customers are equal, we set $\tilde{\Delta}^{H,c}, \tilde{\Delta}^{L,c} = 1$ and compute the volatility of supplier's assets: it is 11.6% under policy (a) while it increases to 16% under (b). This result has an intuitive explanation: the probability that the L-customer is inactive in the downstream market is larger when the supplier adopts a cannibalization of sales policy, it is 0.46% for policy (a) and 0.66% for policy (b). When the L-customer is driven out of the market, the presence of the 'granular' H-customer alone increases the delta-adjusted Herfindahl index, leading to a larger volatility. The straightforward multivariate extension of this result would show that downstream competition and optimal allocation of trade credit by the supplier provides a robust micro foundation also to the second stylized feature documented in Herskovic et al. [2020]: large firms, here proxied by the supplier are less volatile because they are connected to more customers, which improves diversification. In a nutshell, the model provides a structural foundation for the two assumptions that underly the granular volatility approach to the analysis of firm networks of Herskovic et al. [2020], and explains the emergence of a power law distribution of weights and the factoring network statistics that are documented in Subsection 2.1.

4.3 Pricing Implications

Next, we analyze the pricing implications. In our baseline model, bank and trade credit factoring equilibrium prices are determined by the spreads δ^b and δ^{tc}. The computation of bank loan pricing in the model is similar to the benchmark model of Gornall and Strebulaev [2018].

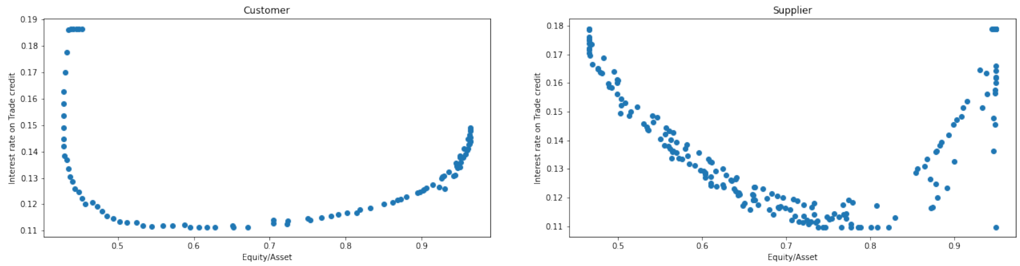


(a) Customer

(b) Supplier

Figure 7: Relation between interest rate on bank credit and equity-to-asset ratio.

Fig. 7 shows our benchmark calibration. Due to the diversification of bank loan portfolio, the optimal equilibrium credit allocations generates a spread δ^b close to 2.5% and



(a) Customer

(b) Supplier

Figure 8: Relation between interest rate on trade credit and equity-to-asset ratio.

virtually independent from customer's and supplier's levels of capitalization. Amberg, Jacobson and Von Schedvin (2021) provide empirical evidence on trade credit and factoring trade credit pricing. In particular, they document the evidence on an annualized 44.6% interest rate implicit in the well-known "2/10 net 30" two-part terms contract and factoring discounts that are currently in the range of 2.5% in Sweden. In the case of the widely used net-30 contracts featuring a $2\% - 5\%$ for the '30-day credit period and no discount option' this corresponds to an implicit annualized interest rates of $24.6\% - 62.4\%$. High rates are also implicit in the statistics produced in Italy by the periodic review conducted by Bank of Italy, for the years relevant to our investigation. Remarkably, Fig.8 documents the ability of our model to match quantitatively the size of factored trade credit. In particular, within the calibrated version of the model, the fair spread charged for a trade credit transaction on an annual basis ranges in the interval $20\% - 35\%$ capturing the documented abnormal values with a convex dependence w.r.t the customer and supplier capitalization. The key advantage provided by the trade credit, that drives the differential pricing with respect to bank credit, is the additional insurance value coming from a flexible repayment conditions offered by suppliers to customers. It is worth observing that in our sample, this flexibility is also supported by a significantly higher recourse to factored trade credit by constrained capital suppliers.

5 Conclusions

We propose a model that jointly determines the capital structure and trade credit factoring allocations of firms, alongside a bank that extends loans to these firms within a supply chain. In addition to the standard bank credit, the firms are inter-connected through a trade credit factoring network facilitating production but also propagating risk along the financial supply chain. We show that when a supplier offering trade credit is less financially constrained, the customers substitute the bank credit in favor of trade credit. Our model, which includes a representative bank, a supplier, and a customer, predicts the empirically observed large cost of factoring due to the additional insurance value coming from flexible repayment conditions that the supplier offers to its customer. We further extend the model to incorporate differential bargaining power among firms, revealing that customers with higher bargaining power demand more trade credit in equilibrium than those with less bargaining power. Suppliers implement contingent trade credit policies ,financed by factoring, to establish relationships with more customers, aiming to maximize revenues while minimizing risk exposure to granular customers.

We provide cross-sectional evidence by analyzing a proprietary dataset of a major Italian bank that offers loans to SMEs and supports trade credit allocations through a factoring facility. The data reveals a star-shaped network of trade credit with suppliers at the center and customers at the end nodes. The empirical analysis of the dataset supports the key model predictions.

There are compelling policy reasons to adopt a model that considers the strategic interactions among firms within a supply chain. Consider, for example, the public intervention programs aimed at sustaining the European production system that was disrupted first by sovereign debt crisis, and more recently, by the pandemic shock. Their ability to improve firm's resilience while preparing the restart as soon as the crises eases, has been often questioned. This proposed modeling approach adds value by enhancing conventional risk management techniques without overlooking the interconnected productive and financial constraints responsible for generating fluctuations in firm volatilities. Ultimately, it can assist financial institutions and policymakers in identifying externalities stemming from trade credit relationships among the borrowers. We leave the policy analysis to future research.

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A Appendix

A.1 Data description and dataset construction

1. **Balance sheet information**: The balance sheet information is provided by Centrale Bilanci and comes at annual frequency. Each item is winsorized at 5th and 95th percentiles respectively. After data cleaning[10], a total of 62,795 firms are retained. While the available balance sheet information is exhaustive, we retain only relevant items that are used in the empirical study.

2. **Internal Rating**: The bank uses an internal categorical credit rating for all its customers. The rating information is on a numerical scale of 2-14 with large number indicating lower rating. The information is available at a monthly frequency.

3. **Credit lines:** The amount of credit line issued and used by the firms is retrieved along with the date of initiation and termination that are available at high frequency. Out of the 62,795 firms with balance sheet information, around 7,600 firms have been issued credit line between the three years.

4. **Loan outstanding:** The loan products of bank include short term and medium/long term loans issued to the customers recorded along with the initiation and termination date. In the database around 33,000 firms have loans outstanding with the bank.

5. **Credit registry:** The Bank of Italy provides information regarding the total use of credit by firms. Each intermediary reports to the central bank the total amount of receivables open to its customers and the central bank in turn provides the bank information about the total indebtedness of each firm in the economy.

6. **Factoring:** A total of 1.66 million factoring transactions over the three years are available at high frequency, aggregated at quarterly or annual basis for the empirical

[10] In addition to winsorizing, data cleaning includes removing firms with missing values and unreasonable values that may be due to data entry error.

study. Each transaction is characterized by a creditor ID, a debtor ID, and the total amount of factoring that is transacted between the parties, providing a novel factoring network among the customers of the bank.

7. **Cash transaction:** The cash transaction is also characterized by a creditor ID, a debtor ID, and the total amount transacted through wire. The total number of transactions in which both creditors and debtors are customers of the bank are 911,775 over the three years. As opposed to factoring information in which both creditors and debtors are customers of the bank, cash transaction data covers also inflows and outflows to and from firms who are not the customers of the bank as well. The total number of transactions amounts to a total of 23.77 million edge data points.

A.2 Network descriptive statistics

We construct the trade credit network using the factoring transactions intermediated by the bank. The factoring data determine the weight of each edge where there is a creditor, a debtor, and the amount transacted between the parties. Formally, the data determine a directed network $\mathcal{G}$ that is defined by the pair of sets $(V^{\mathcal{G}}, E^{\mathcal{G}})$ along with the adjacency matrix $A^{\mathcal{G}}$ with individual elements $w_{i,j} \geq 0$ denoting weights for the pair $(i, j) \in E^{\mathcal{G}}$. The vertices $V^{\mathcal{G}}$ represent the firm-specific node information and the edges $E^{\mathcal{G}}$ represent the information pertaining to the transactions between two parties on either side of the edges. The weights $w_{i,j}$ are computed from the credit amount that is transferred between the firms i and j. That is,

$$w_{i,j} = \frac{FC_{i,j}}{\sum_k FC_{k,j}} \tag{7}$$

where $FC_{i,j}$ is the amount of trade credit that firm i, as a creditor, offers to the firm j, as debtor. Thus, the weights measure the relative importance of this type of credit with respect to the overall credit issued. In the view of a supply-chain financing model, the creditors act as suppliers and the debtors act as the customers, hence we use these terms

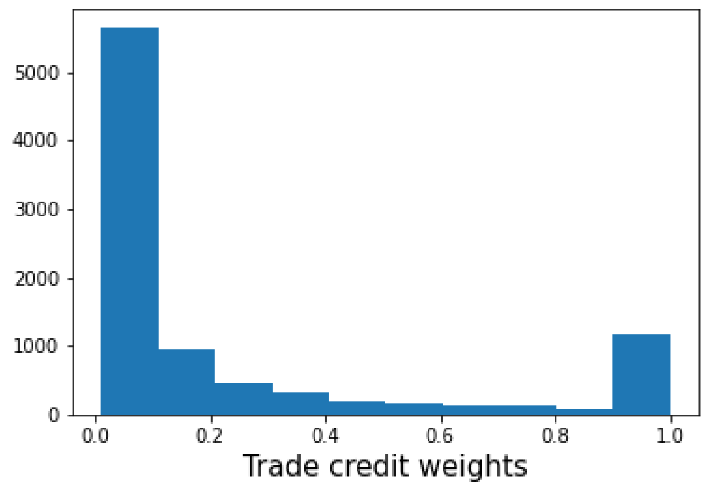


Figure 9: Histogram of factored credit weights constructed using (7).

interchangeably. Note that $w_{i,j}$ is different from $w_{j,i}$ generating a directed network with an asymmetric adjacency matrix. The data is available at high frequency but, for the empirical study, we aggregate all amounts transacted between the parties at annual frequency. Figure (9) plots the distribution of weights $w_{i,j}$ in logarithmic scale. As shown in Table (7), a power-law fit to the empirical distribution reveals an *alpha* in the range 1.5-1.65 indicating heavy tails, which can be corroborated by visualizing the plot in Figure (9). The spike in the weights at value 1.0 is due to the fact that around 5% of the firms in the database have only one customer and by construction, the weight is equal to 1. Interestingly, the third row in Table (7) shows that even if the network is trimmed such that suppliers with only one customer are removed from the database, the weights still exhibit heavy tails showing that the fat-tailedness is a feature of the network that is robust to outliers.

	2013	2014	2015
Alpha (Weights)	1.50	1.49	1.48
Alpha (Weights trimmed)	1.65	1.65	1.61

Table 7: Power law exponent (alpha) of adjacency matrix weights. The third row presents the fit to weights with values equal to 1 removed.

	Mean	Std	Min	Max
Total Assets	26014.07	18010.34	227.00	48514.75
Leverage	7.60	7.84	1.00	39.00
Fixed Assets/Total Assets	0.29	0.16	0.00	0.99
log Assets	9.76	1.07	5.42	10.79
Liquidity	932.49	896.15	0.00	2193.75
Liquidity/Total Assets	0.04	0.05	0.00	0.40
Ratings	7.60	2.66	2.00	14.00

Table 8: Supplier statistics across all three years. Number of unique suppliers is 756.

	Mean	Std	Min	Max
Total Assets	38924.71	16323.18	189.00	48514.75
Leverage	5.59	5.88	1.00	39.00
Fixed Assets/TotalAssets	0.34	0.11	0.00	0.83
log Assets	10.32	0.96	5.24	10.79
Liquidity	1463.17	928.94	0.00	2193.75
Liquidity/Total Assets	0.04	0.04	0.00	0.45
Ratings	6.17	2.73	2.00	14.00

Table 9: Customer statistics across all three years. Number of unique customers is 9162.

	Component 1	Component 2	Component 3	Remaining Components	All firms in Network
Liquidity	941.61	1085.76	409.49	903.26	907.36
Total Assets	26943.39	32839.15	11363.15	25846.21	26063.95
Sales Growth (%)	2.75	8.88	3.36	3.76	3.43
ROA	5.03	2.44	3.72	4.90	4.88
Leverage	5.75	6.64	7.04	6.85	6.42
Liquidity Ratio	0.04	0.04	0.05	0.04	0.04
Rating	6.42	7.83	7.94	7.20	6.90
Equity/Assets	0.28	0.76	0.41	0.27	0.29
Accounts Receivables/Assets (%)	0.58	0.68	0.42	0.59	0.58
Operating Cost/Assets (%)	2.16	2.31	1.59	2.23	2.18
Fixed Assets/Assets	0.28	0.31	0.25	0.28	0.28

Table 10: Comparison of key variables across first three components and all firms for the year 2014. *in thousands of euro. All quantities refer to the average value.

	Component 1	Component 2	Component 3	Remaining Components	All firms in Network
Liquidity	1053.76	1001.21	474.00	944.21	977.91
Total Assets	29045.41	31825.32	12454.11	25706.33	26861.16
Sales Growth (%)	5.06	9.01	10.51	5.73	5.55
ROA	5.64	3.77	4.34	5.21	5.34
Leverage	5.21	6.70	7.84	6.84	6.21
MOL	1685.04	1744.09	653.31	1526.74	1574.56
Liquidity Ratio	0.05	0.04	0.05	0.05	0.05
Rating	7.81	8.68	8.14	6.70	6.75
Equity/Assets	0.31	0.53	0.24	0.27	0.29
Accounts Receivables/Assets (%)	0.58	0.71	0.43	0.57	0.57
Operating Cost/Assets (%)	2.28	2.33	1.67	2.24	2.23
Fixed Assets/Assets	0.29	0.32	0.25	0.28	0.28

Table 11: Comparison of key variables across first three components and all firms for the year 2015. *in thousands of euro. All quantities refer to the average value.

B The model

For simplicity, we assume that bank's and firm's strategic decisions and payments take place in two periods. At time $t = 0$ a customer c borrows from a bank and the loan is to be repaid at time $t = T$. The customer also borrows in the form of trade credit from a supplier. If the customer is unable to repay both the bank as well as the supplier, then she becomes insolvent and her assets are liquidated. We assume that the bank loan repayment is senior in case of a default. At $t = T$, if the customer is able to repay the bank, but not the supplier, she can exercise an option to delay the repayment of the amount due to the supplier to time $t = 2T$, to avoid insolvency. The customer hopes to receive a positive cash flow shock between $t = T$ and $t = 2T$ to be able to repay in full the amount due to the supplier and avoid bankruptcy. We solve the problem by backward induction starting from time $t = 2T$. Let us consider the individual positions of a representative customer, of a supplier, and of the bank.

B.1 The customer problem

B.1.1 Time $t = 2T$

The asset growth equation for the customer at $t = 2T$ is given by

$$\log\left(\frac{A^c_{2T}}{A^c_T}\right) = -\frac{1}{2}T\sigma_c^2 + \sigma_c \hat{Z}^c_T \tag{8}$$

where $\hat{Z}^c_T$ is a normal shock with variance T, σ is the volatility parameter, and the maturity of the loan is T. The promised repayment amount to the supplier is denoted by $R^c_{\mathcal{F}}$ and its value at time $t = 0$ is $V^c_{\mathcal{F}}$. If the asset value A^c_{2T} falls below a threshold amount $\hat{\zeta}^c$, then the customer defaults on the supplier and becomes insolvent, eventually. We assume that between period T and $2T$, the customer does not receive a new bank loan. The free cash flow of the customer is

$$A_{2T}^{c} - \tau \max\{0, A_{2T}^{c} - (R_{\mathcal{F}}^{c} - V_{\mathcal{F}}^{c})\} \tag{9}$$

where τ is the tax rate and the interest on the amount of trade credit received is tax deductible. If the free cash flow is smaller than the repayment amount $R_{\mathcal{F}}^{c}$, then the firm becomes insolvent at the end of $t = 2T$. This condition pins down the asset threshold value which is given by

$$\hat{\zeta}^{c} = R_{\mathcal{F}}^{c} + \frac{\tau}{1-\tau} V_{\mathcal{F}}^{c} \tag{10}$$

and the quantity V_{T}^{c} denoting the value at time T of the amount that will be repaid at end of $t = 2T$. Then, we have

$$V_{T}^{c} = e^{-r_f} E\left[\mathbb{1}_{A_{2T}^{c} \geq \hat{\zeta}^{c}} R_{\mathcal{F}}^{c} + \mathbb{1}_{A_{2T}^{c} < \hat{\zeta}^{c}} (1-\tau)(1-\alpha) A_{2}^{c} \right] \tag{11}$$

where α is the bankruptcy cost.

B.1.2 Time $t = T$

The asset growth between $t = 0$ and time $t = T$ is given by

$$\log\left(\frac{A_{T}^{c}}{A_{0}^{c}}\right) = -\frac{1}{2} T \sigma_{c}^{2} + \sigma_{c} Z_{T}^{c} \tag{12}$$

Where Z_{T}^{c} is a normal shock with variance T that is independent of the shock $\hat{Z}^{c}$. The customer issues a zero coupon bank debt security valued at V_{D}^{c} with a promised repayment of R_{D}^{c}, and issues a trade credit security to the supplier valued at $V_{\mathcal{F}}^{c}$ with a promised repayment of $R_{\mathcal{F}}^{c}$. Let the total repayment value be denoted as R_{F}^{c}. That is, $R_{F}^{c} = R_{D}^{c} + R_{\mathcal{F}}^{c}$. Assume that both the interest payment on bank debt and on trade credit is tax deductible.

In case of insolvency, the bank gets paid first and receives

$$\min\{R_D^c, (1-\alpha)(1-\tau)A_T^c\} \tag{13}$$

where α is the bankruptcy cost and τ is the tax rate as before. We denote by ζ^c the lower threshold barrier for the asset value A_T^c. If $A_T^c < \zeta^c$ the customer is in distress and, if forced to repay at time $t = T$ all his debt, she would be insolvent on her total debt. However, the customer has the option at time $t = T$ to delay the payment owed to suppliers to time $t = 2T$, provided she can repay the bank credit but not the supplier. This implies that the customer's free cash flow is given by

$$A_T^c - \tau \max\{0, A_T^c - (R_F^c - V_D^c - V_{\mathcal{F}}^c)\} \tag{14}$$

The firm becomes insolvent at $t = T$ only if it is unable to repay the bank. The condition is given by

$$A_T^c - \tau \max\{0, A_T^c - (R_D^c - V_D^c)\} < R_D^c \tag{15}$$

This condition pins down the threshold value ζ^c as

$$\zeta^c = R_D^c + \frac{\tau}{1-\tau} V_D^c \tag{16}$$

and correspondingly the value of the customer's bank debt is given as

$$V_D^c = e^{-r_f} E\left[R_D^c \mathbb{1}_{A_T^c \geq \zeta^c} + \min\{(1-\tau)(1-\alpha)A_T^c, R_D^c\}\mathbb{1}_{A_T^c < \zeta^c}\right] \tag{17}$$

The value of trade credit embeds the option value of postponing the payment to $t = 2T$. There are three possible outcomes. The first scenario is when the firm becomes insolvent i.e., $A_T^c < \zeta^c$ in which case the suppliers get the residual amount $(1-\alpha)(1-\tau)A_T^c - \min\{R_D^c, (1-\alpha)(1-\tau)A_T^c\}$. The second scenario is when the firm is not insolvent ($A_T^c >$

ζ^c) but the customer is unable to repay the supplier. This happens under the following condition

$$A^c_T - \tau \max\{0, A^c_T - (R^c_D - V^c_D)\} - R^c_D < R^c_{\mathcal{F}} \tag{18}$$

Let $\bar{\zeta}^c$ denote the lower asset value threshold triggering the exercise of the option to postpone the payment by the customer to period $t = 2T$. This threshold value can be obtained from the equation (18) as

$$\bar{\zeta}^c = R^c_{\mathcal{F}} + R^c_D + \frac{\tau}{1-\tau} V^c_D \tag{19}$$

Comparing equations (16) and (19), we see that $\bar{\zeta}^c > \zeta^c$ as expected. Then the value of trade credit is given by

$$V^c_{\mathcal{F}} = e^{-r_f} E\left[\mathbb{1}_{A^c_T < \zeta^c} \max\{0, (1-\alpha)(1-\tau)A^c_T - R^c_D\} + \mathbb{1}_{\zeta^c < A^c_T < \bar{\zeta}^c} V^c_T + \mathbb{1}_{\bar{\zeta}^c < A^c_T} R^c_{\mathcal{F}}\right] \tag{20}$$

Similarly, the value of firm equity also depends on whether $\zeta^c < A^c_T < \bar{\zeta}^c$ or whether $\bar{\zeta}^c < A^c_T$. In the latter case, the free cash flow is equal to the asset value net of the repayment amount of both bank debt and trade credit since the customer has enough to repay both. In the former case, the free cash flow is modified as $A^c_T - \tau \max\{0, A^c_T - (R^c_D - V^c_D) - R^c_D\} + F^c_T$ where F^c_T is the discounted expected value of free cash flow from time $t = 2T$ given as

$$F^c_T = e^{-r_f} E\left[A^c_{2T} - \tau \max\{0, A^c_{2T} - (R^c_{\mathcal{F}} - V^c_{\mathcal{F}})\} - R^c_{\mathcal{F}}\}\right] \tag{21}$$

Thus, the equity value is given as

$$\begin{aligned} V^c_E = e^{-r_f} E\Big[& (A^c_T - \tau \max\{0, A^c_T - (R^c_F - V^c_D - V^c_{\mathcal{F}})\} - R^c_F)\mathbb{1}_{A^c_T > \bar{\zeta}^c} \\ & + (A^c_T - \tau \max\{0, A^c_T - (R^c_D - V^c_D)\} - R^c_D + F^c_1)\mathbb{1}_{\zeta^c < A^c_T < \bar{\zeta}^c}\Big] \end{aligned} \tag{22}$$

B.2 The supplier problem

Assume that a supplier s is connected by a star network to a number N^s number of customers through trade credit flows financed by the factoring service. The trade-credit amount to customer c is priced at $V^c_{\mathcal{F}}$ with a promised repayment amount of $R^c_{\mathcal{F}}$. The total value of the trade credit to the supplier is simply the sum of the trade credit values issued to its customers. That is,

$$V^s_{\mathcal{F}} = \sum_{j=1}^{N^s} V^c_{j\mathcal{F}} \tag{23}$$

where $V^c_{j,\mathcal{F}}$ denotes the value of trade credit issued by the supplier s to the individual customer j. Let the payoff from the trade credit issued to the customer j be denoted by Π^c_j. Then,

$$\Pi^c_j = \mathbb{1}_{A^c_{j,T}<\zeta^c_j} \max\{0, (1-\alpha)(1-\tau)A^c_{j,T} - R^c_D\} + \mathbb{1}_{\zeta^c_j<A^c_{j,T}<\bar{\zeta}_j{}^c} F^c_{j,T} + \mathbb{1}_{\bar{\zeta}_j{}^c<A^c_{j,T}} R^c_{j,\mathcal{F}} \tag{24}$$

where $R^c_{j,D}$ is the promised repayment amount of the customer to the bank, the thresholds $\zeta^c_j, \bar{\zeta}^c_j$, and $F^c_{j,T}$ are given in the customer problem in the previous section. The asset equation of the supplier can be written in the differential form as

$$\frac{dA^s_t}{A^s_t} = \gamma \sum_{j=1}^{N^s} w_j \tilde{\Delta}^c_{j,t} \frac{dA^c_{j,t}}{A^c_{j,t}} + \sigma_s dZ^s_t - l dH_t \tag{25}$$

where Z^s is s standard Brownian shock, H_t is a binary random variable that takes value 1 with intensity λ_s and 0 otherwise, and l is a constant jump size. The variable H_t captures the liquidity risk, which happens occasionally with probability λ_s.

The first component, on the r.h.s. represents the growth opportunities emerging from the trade credit relationship. The variable $\tilde{\Delta}^c_j = \frac{A^c_j}{\Pi^c_j}\frac{d\Pi^c_j}{dA_j}$ is the elasticity of the composite option value with respect to the underlying, where Π_j denotes the value of the composite option whose dynamics are given by equation (24). Details about its computation are given in Prop. (7) in this Appendix. An adverse liquidity event ($\lambda_s > 0$) reduces profitability (the growth rate of the assets). By stipulating a factoring contract with the bank, the supplier improves the liquidity position that is captured by the second component on the r.h.s. of the equation:

$$\log A_T^s = \log A_0^s + \left(\gamma \sum_{j=1}^{N^s} w_j \int_0^1 \tilde{\Delta}_{j,s}^c \frac{dA_{j,s}^c}{A_{j,s}^c} \right) - \frac{1}{2} T\sigma_s^2 + \sigma_s Z_T^s + H_T \log(1-l) \tag{26}$$

When $H_T = 1$, the value of assets gets reduced by a proportion l since $l < 1$.

Note that equation (26) introduces the key network interaction, assets of suppliers are exposed to customer's asset shocks weighted by the proportion of trade credit amount transacted between the two parties, and parameter γ captures the strength of the network propagation effect. The interpretation of γ is similar to Herskovic et al (2020). A value close to zero indicates no network effect, whereas a value that is close to 1 indicates a strong network effect. The presence of customer trade credit increases the volatility of supplier assets by an amount $\gamma^2 \sum_c \left(\tilde{\Delta}_T^c w_j \sigma_j^2 \right)^2 = \gamma^2 H_{out}$ where H_{out} is the Delta-volatility-adjusted Herfindahl-out index for the supplier s. Note that the value of the individual weight will depend on the industry-specific relationship between the customer the and supplier, while the delta contribution accounts for the impact of the customer's financial riskiness.

The supplier raises equity in the competitive capital market to finance its operations, and debt in the form of credit from the bank and offers a trade credit facility to customers. In particular, the supplier raises V_D^s in the form of a loan with a face value of R_D^s. The amount of trade credit is in the form of accounts receivables, which means that the customer promises to pay the supplier the amount $R_{\mathcal{F}}^c$ in the future for the borrowed trade credit amount priced at $V_{\mathcal{F}}^c$. The free cash flow of the supplier is given by

$$A_T^s - \tau \max\{0, A_T^s - V_{\mathcal{F}}^s - (R_D^s - V_D^s)\} \tag{27}$$

where $A_T^s - V_{\mathcal{F}^s}$ is the tax base, and $R_D^s - V_D^s$ is the interest paid on bank debt. The supplier defaults on the bank loan if and only if the free cash flow is lower than the promised repayment amount R_D^s. The asset A_T^s threshold value to default is then:

$$\zeta^s = R_D^s + \frac{\tau}{1-\tau}(V_D^s - V_{\mathcal{F}}^s) \tag{28}$$

and the value of debt and equity is

$$V_D^s = e^{-r_f} E[R_D^s \mathbb{1}_{A_T^s \geq \zeta^s} + (1-\alpha)(1-\tau)A_T^s \mathbb{1}_{A_T^s < \zeta^s}] \tag{29}$$

$$V_E^s = e^{-r_f} E[(A_T^s - \tau \max\{0, A_T^s - (R_D^s - V_D^s) - V_{\mathcal{F}}^s\})\mathbb{1}_{A_T^s \geq \zeta^s}] \tag{30}$$

The trade credit spread δ^{tc} is determined taking the option value into account as follows. That is, δ^{tc} satisfies $V_{\mathcal{F}}^s = \tilde{V}_{\mathcal{F}}^s$, where

$$V_{\mathcal{F}}^s = e^{-(r_f+\delta^{tc})T} N_c E[\mathbb{1}_{A_T^c < \zeta^c} \max\{0, (1-\alpha)(1-\tau)A_T^c - R_D^c\} + \mathbb{1}_{\zeta^c < A_T^c < \bar{\zeta}^c} \hat{V}_1^c + \mathbb{1}_{\bar{\zeta}^c < A_T^c} R_{\mathcal{F}}^c] \tag{31}$$

$$\tilde{V}_{\mathcal{F}}^s = e^{-r_f T} N_c E[\mathbb{1}_{A_T^c < \bar{\zeta}^c} \max\{0, (1-\alpha)(1-\tau)A_T^c - R_D^c\} + \mathbb{1}_{\bar{\zeta}^c < A_T^c} R_{\mathcal{F}}^c] \tag{32}$$

B.3 The bank problem

The bank issues loans to both the suppliers and the customers who use their respective cash flows A_T^s and A_T^c as collateral with promised repayment of R_D^s and R_D^c. The bank receives the promised repayment as long as the collateral is above the respective thresholds ζ^c and ζ^s. In case of default, the collateral is taken over by the bank, after the bankruptcy cost α_b.

The bank borrows a nominal amount of B from competitive debt and equity markets to issue loans to the firms. The promised repayment on the zero coupon bond that the bank borrows is R^b, and the interest paid on the debt is tax deductible. The free cash flow is given by

$$B - \tau \max\{0, B - V_A^b - (R^b - V_D^b)\} \tag{33}$$

The bank defaults if the free cash flow is below the promised payment on the debt security. That is, the default condition is

$$B - \tau \max\{0, B - V_A^b - (R^b - V_D^b)\} < R^b$$

Since $V_A^b > V_D^b$, this default condition simplifies to $B < R^b$. That is, the bank defaults whenever the payoff on loans is lower than the promised repayment amount. Since the banks operate in a competitive market, the spread δ^b is determined such that the bank makes zero profit. In conclusion, the value of bank debt and equity is then given by

$$V_D^b = e^{-r_f} E\left[R_B \mathbb{1}_{B \geq R_B} + (1 - \alpha_b) B \mathbb{1}_{B < R_B}\right] \tag{34}$$

$$V_E^b = e^{-r_f} E\left[\left(B - \tau \max\{0, B - V_A^b - R_B + V_D^b\} - R_B\right) \mathbb{1}_{B \geq R_B}\right] \tag{35}$$

Notice that the final effective aggregate payoff produced by the bank loan portfolio will in general depend on the overall configuration of the network. In fact, any shock to the customers that affect the suppliers' assets transfers directly into the bank assets. Institutionally, the bank must implement a risk mitigation policy by selecting a lending policy that minimizes the exposures to firm-specific shocks. The resulting optimization problem entails the analysis of the systemic risks addressed by bank risk managers and is in general complicated by the presence of an entire web of interactions and of correlations among firm cash flows.

Since the main focus of this paper is on the impact of trade credit relationships, we consider a reduced-form problem assuming that, at least at the level of direct bank lending, portfolio exposure to firm-specific shocks are removed. This is equivalent to assuming a default condition where the payoff B is replaced by its conditional expected value

$B(Y) := E[B|Y]$. Notice that this averaging impacts only the bank default condition, while the price of the loan, conditional on no default, is still dependent on the realization of all shocks and the network configuration. Hence, diversification at the level of an individual supplier depends on the concentration of trade credit positions, and the shocks in the network component Ω will not be fully diversified. In particular, the volatility of the bank assets is increasing in the delta-volatility-adjusted Herfindhal-out index $\gamma^2 H_{out}$.

Therefore, the total payoff of the bank loans issued to suppliers s or customers c (for notational simplicity we consider the basic case with one suppliers)

$$
\begin{aligned}
B &:= B^c + B^s \\
&= (R^c_D \mathbb{T}_{A^c_T \geq \zeta^c} + (1-\alpha_b) A^c_T \mathbb{1}_{A^c_T < \zeta^c}) + (R^s_D \mathbb{1}_{A^s_T \geq \zeta^s} + (1-\alpha_b) A^s_T \mathbb{1}_{A^s_T < \zeta^s})
\end{aligned}
$$

For a given initial level of assets A_0, the final value of total collateral taking into account the standard normally distributed shock Y is given by

$$
\log\left(\frac{A^c_T}{A^c_0}\right) = \sqrt{\rho} Y + \sqrt{(1-\rho)}\sigma_c Z^c_T - \frac{1}{2}\sigma_c^2 T \tag{36}
$$

$$
\log\left(\frac{A^s_T}{A^s_0}\right) = \left(\gamma \sum_j w_{s,j} \int_0^1 \tilde{\Delta}^{c,j}_s \frac{dA^{c,j}_s}{A^{c,j}_s}\right) + Y + \sqrt{(1-\rho)}\sigma_s Z^s_T - \frac{1}{2} T \sigma_s^2 + H \log(1-l) \tag{37}
$$

where a parameter ρ is added to control the average correlation between the idiosyncratic shocks (Z^c_T and Z^s_T) and the systematic shock Y. We assume that $Y \sim N(0, T)$ and independent of the other shocks.

B.4 Modeling The interaction between financial and production decisions.

Eq.(25) states that the risks driving customer asset dynamics reverberate in supplier's assets dynamics with an intensity that depends on an exogenously specified weight w_j multiplied by a measure of customer risk exposure $\tilde{\Delta}^c_j$ accounting for the individual customer leverage and counterparty risk.

The weight w_j quantifies the relative importance of a customer within the portfolio of customers that are served by supplier's production. The supplier decision to allocate production and trade credit discounts across different customers has been the subject of a thorough empirical and model based analysis. In particular, Giannetti et al. [2021] provide evidence that the suppliers implement a sale strategy on quantities and the trade credit contract terms aimed at easing competition in the downstream market. Hence, in equilibrium, weights w_j are an outcome of an endogenous decision process that affects the production capacity of customers competing in the downstream market, above and beyond the financial riskiness of the relevant stakeholders across the chain.

In order to quantify the trade-offs faced by customers and suppliers to cope simultaneously with financial and production constraints, we consider a supply chain where the bank and the supplier serve two customers $j = H, L$ with differential bargaining power that compete a la Cournot in the downstream market. Then the selection of the weights w_j that takes place at time 0, immediately after a demand shock is realized, can be endogenized embedding the structural modeling approach of bargaining between customers and suppliers formalized by Giannetti et al. [2021].

We denote by $q^H(\phi, \alpha)$ the input demand function from the H-customer conditional on a demand shock α and on ϕ, which is the discount over firm H cost of capital. Then the supplier can choose the terms of the trade credit contract $(\phi, \overline{x})$ that identify respectively the size of the unit discount ϕ and the maximum dollar amount $\overline{x}$ of trade credit offered with a discount with respect to the borrower's cost of capital. Trade credit offer has to satisfy firm H participation constraint in expectation, that can be written as:

$$\phi \int_{\underline{\alpha}}^{\alpha^{**}} q^H(\phi, \alpha) dF_\alpha = \bar{U}. \tag{38}$$

where F_α denotes the cumulative probability distribution function of the demand shock

$\alpha > 0$ over the support $[\underline{\alpha}, \bar{\alpha}]$. The parameter α^{**} is set by the condition

$$q^H(\phi, \alpha^{**}) = \frac{\overline{x}}{m}$$

where m is the unit price of input good that is financed by the trade credit position and $q^H(\phi, \alpha)$ denotes the maximal demand that can be financed relying on the factoring facility.[11] Since the two firms are ex-ante identical and differ only in their ability to bargain with the supplier, we proxy the common input cost m considering the average equilibrium cost of capital provided by the factoring service in the absence of markups. That is, we set $m = \frac{1}{2Q^P} E_0[(R^H_{\mathcal{F}} + R^L_{\mathcal{F}} - V^H_{\mathcal{F}} - V^L_{\mathcal{F}})]$, where Q^P is the planned quantity.[12] In this way, the marginal cost for the H- and L-customer are equal ex-ante and the only effective difference is determined by the discount ϕ that the supplier applies only to the H-customer. We set the planned quantity $Q^P = 1$.[13]

We follow Giannetti and we assume that, given the unit price of m, the supplier offers a contract to firm H, granting surplus U . Once, the demand shock realizes firm L decides whether to enter paying a cost K. Then, firms simultaneously choose the quantity q^H and q^L. Consider first the case in which the supplier does not take into consideration the impact of her sale policy on downstream market competition and sets a sale to H-customer offering a discount ϕ on any amount of trade credit borrowed. Then, a standard Cournot competition argument implies the following equilibrium trade credit quantities.

[11] In addition, it is assumed that $\underline{\alpha} \geq m + \phi$ and $\phi < m$. See Gianetti et al 2021 for details.

[12] Note that the model implications remain the same if we use the expected total quantity instead of planned quantity Q^P. However, assuming planned quantity simplifies the numerical computation without altering the main conclusions of the analysis.

[13] Note that in the calibrated model with a large realized demand shock, the endogenous total quantity obtained is $Q = 1.14$.

Proposition 3 *The quantities of trade credit demanded by the customers are given by:*

$$w^H = \frac{q^H}{q^H + q^L}; \qquad w^L = 1 - w^H$$

where

$$q^H(\phi, \alpha) = \begin{cases} \frac{1}{2} * (\alpha + \phi - m) & \alpha \leq \alpha^*(\phi, K) \\ \frac{1}{3} * (\alpha - m + 2\phi) & \alpha > \alpha^*(\phi, K) \end{cases} \tag{39}$$

and

$$q^L(\phi, \alpha) = \begin{cases} 0 & \alpha \leq \alpha^*(\phi, K) \\ \frac{1}{3} * (\alpha - m - \phi) & \alpha > \alpha^*(\phi, K) \end{cases} \tag{40}$$

Proof: See Section D.

Note that below the threshold $\alpha^*(\phi, K) := m + 3\sqrt{K} + \phi$ the L-customer optimal choice is not to produce and withdraw from the downstream market.

In this case, this discount policy implies a cannibalization of sales for the supplier that would not sell to the L-customer when the demand shock is not large enough. In fact, everything else fixed, an increase of discount ϕ increases the likelihood of L-customer being driven out of the downstream market due to the expansion of sales of the H-customer when the demand shock is below the threshold $\alpha^*(\phi, K)$. In order to minimize the ex-ante risk of cannibalization, the supplier can offer a discount ϕ only on a limited quantity, up to a dollar value of goods sold smaller or equal to $\bar{x}$. By setting $\phi = 0$ for high levels of demand shock, the new quantities demanded by the customers are given in the following:

Proposition 4 *The quantities of trade credit demanded by the customers are given by:*

$$w^H = \frac{q^H}{q^H + q^L}; \qquad w^L = 1 - w^H$$

where

$$q^H(\phi, \alpha) = \begin{cases} \frac{1}{2} * (\alpha + \phi - m) & \alpha \leq \alpha^*(\bar{x}, K) \\ \frac{1}{3} * (\alpha - m) & \alpha > \alpha^*(0, K) \end{cases} \tag{41}$$

and

$$q^L(\phi, \alpha) = \begin{cases} 0 & \alpha \leq \alpha^*(\bar{x}, K) \\ \frac{1}{3} * (\alpha - m) & \alpha > \alpha^*(0, K) \end{cases} \tag{42}$$

Proof. Set $\phi = 0$ when $\alpha > \alpha^*(\phi, K)$ in Proposition 3. ■

Proposition 5 characterizes the optimal policy that the supplier chooses to avoid distortions in the downstream market.

Proposition 5 *By offering a maximum dollar value $\overline{x} \leq m\sqrt{K}$ and a discount ϕ that satisfies the condition (38), the supplier avoids competition distortions in the downstream market.*

Proof. See Section D. ■

In summary, the following outcomes are relevant to our discussion: assume first that the supplier chooses $\overline{x} \leq m\sqrt{K}$ to avoid risk of cannibalization of sales to the L-customer. In this case, the demand functions are given by equations (41) and (42).

Then, depending on the realization of the shock, the following scenarios may occur:

1. *Small demand shock asymmetric equilibrium* The limit $\overline{x} < m\sqrt{K}$ and $\alpha < \alpha^*$. The supplier chooses a policy to avoid cannibalization of the L-customer when the realized demand shock is small.

2. *Large demand shock, symmetric equilibrium*: The limit $\overline{x} < m\sqrt{K}$ and $\alpha > \alpha^*$. The supplier chooses a policy to avoid cannibalization of L-customer sales, and the

realized demand shock is large. In this case, the equilibrium is symmetric and $w_H^* = w_L^* = \frac{1}{2}$.

As a counterfactual, consider the case $\overline{x} > m\sqrt{K}$ and suppose that the supplier chooses a suboptimal policy that does not avoid cannibalization in downstream markets. Then demand functions of the customers are then given by:

Proposition 6 *The quantities of trade credit demanded by the customers are given by:*

$$w^H = \frac{q^H}{q^H + q^L}; \qquad w^L = 1 - w^H$$

$$q^H(\phi, \alpha) = \begin{cases} \frac{1}{2} * (\alpha + \phi - m) & \alpha \leq \underline{\alpha}^{**}(\bar{x}, K) \\ \frac{1}{3} * (\alpha + 2\phi - m) & \underline{\alpha}^{**}(\bar{x}, K) < \alpha < \bar{\alpha}^{**}(\bar{x}, K) \\ \frac{1}{3}(\alpha - m) & \bar{\alpha}^{**}(\bar{x}, K) < \alpha \end{cases} \tag{43}$$

and

$$q^L(\phi, \alpha) = \begin{cases} 0 & \alpha \leq \underline{\alpha}^{**}(\bar{x}, K) \\ \frac{1}{3} * (\alpha - \phi - m) & \underline{\alpha}^{**}(\bar{x}, K) < \alpha < \bar{\alpha}^{**}(\bar{x}, K) \\ \frac{1}{3}(\alpha - m) & \bar{\alpha}^{**}(\bar{x}, K) < \alpha \end{cases} \tag{44}$$

Proof: Follows from Propositions 3 and 4.

where the threshold $\underline{\alpha}^{**}$ is the threshold below which the L-customer decides not to enter the market. It is given by $\underline{\alpha}^{**} = m + \phi + 3\sqrt{K}$. The value $\bar{\alpha}^{**}$ denotes the threshold below which the supplier provides trade credit at a discount. It is given by $\bar{\alpha}^{**} = m - 2\phi + 3\overline{x}/m$. Then the demand functions are given in (43)and (44) and the following outcomes will realize:

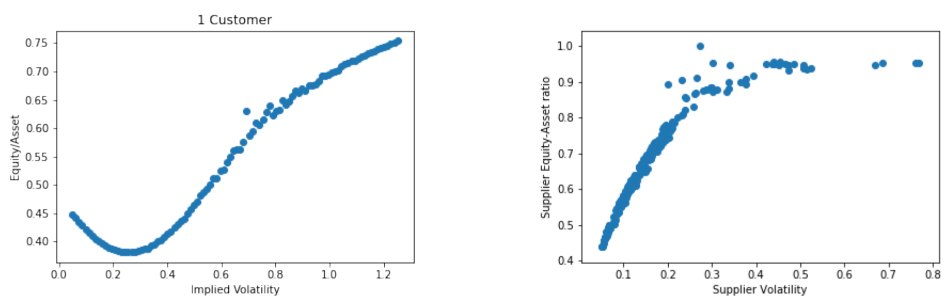
1. *Small demand shock, asymmetric equilibrium*: The limit $\overline{x} > m\sqrt{K}$ and $\alpha < \underline{\alpha}^{**}$. The supplier chooses a policy with risk of cannibalization of sales to the L-customer, and

the realized demand shock is small. In this case, only the H-customer will be active in downstream market.

2. *Intermediate demand shock, asymmetric equilibrium*: The limit $\overline{x} > m\sqrt{K}$ and $\underline{\alpha}^{**} < \alpha < \bar{\alpha}^{**}$. For the intermediate levels of demand shock, both customers will be active in the downstream market, but they choose different quantities.

3. *Large demand shock, symmetric equilibrium*: The limit $\overline{x} > m\sqrt{K}$ and $\bar{\alpha}^{**} < \alpha$. In this case, the demand shock is large and both the customers are active in the downstream market, choosing the same quantities $w_H^* = w_L^* = \frac{1}{2}$.

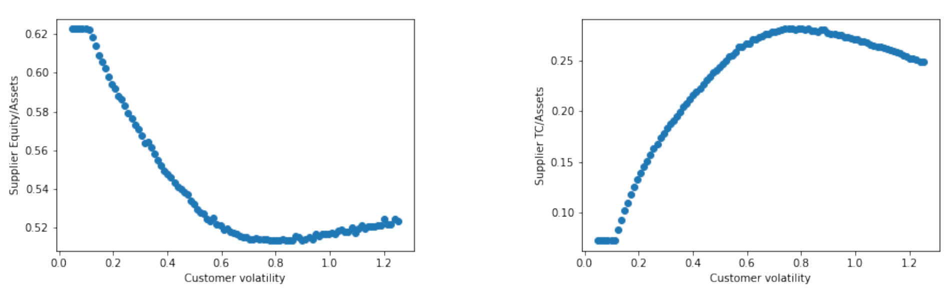
C Sensitivity with respect to the exogenous parameters



(a) Customer Equity/Asset against its implied volatility.

(b) Supplier's equity-asset ratio against its asset implied volatility.

Figure 10: Customer and supplier capital structures against their asset implied volatilities



(a) Supplier's Equity/Asset against customer volatility

(b) Supplier's Trade credit/Asset against customer volatility

Figure 11: Impact of customer asset implied volatility on supplier capital structure.

In order to assess the overall quality of the extension of the Gornall and Strebulaev [2018] model, we run a preliminary analysis of the sensitivity of our findings with respect to exogenous parameters and verify that the model of supply chain credit captures important documented stylized facts. We consider the case with a single customer ad confirm that for both customers and suppliers the model produces an endogenous inverse relationship between the level of asset riskiness and the observed level of leverage. Firms with riskier assets set a lower level of equilibrium leverage for a given level of bankruptcy costs. We reproduce it for both customers and suppliers in Fig. 10 where we chart equity capitalization as a function of implied asset volatilities. In order to generate a realistic distribution of the risk-driving customer and supplier assets, for each firm in the sample, we compute the level of asset volatility implied by the individual default probability as given by the Altman Z-score, following the conventional reconstruction procedure implied by the use of the Merton structural model and of the Black and Scholes formula.

The model provides information sufficient to run a strategic equilibrium analysis of the two competing forms of debt: bank credit and a factoring service, with this last one offering customers also additional insurance with respect to liquidity shocks.

In Fig. 11 we show that the solution highlights a (mild) decreasing relationship between the capitalization of the *supplier* and of the *customer* and their asset risk, confirming that the factoring relationship drives inter-firm propagation of risks. Correspondingly, we chart also the relationship between the size of the trade credit position as a function of the customer's asset-implied volatility. The result is hump-shaped. Below a threshold level of asset risk (around 70% in our calibration), an increase in customer's asset risk drives a decrease in the level of capitalization of the supplier and an increase in the amount of trade credit supplied to the customer. In fact, a large value of customer volatility implies more revenues from trade credit flow: hence, as the customer volatility increases, the supplier will borrow more from the bank reallocating the benefit of the debt to the customer by raising the amount of trade credit financed wit factoring. Beyond the threshold risk level, increasing customer volatility is harmful because the volatility of the supplier's collateral becomes so large to push the supplier towards bankruptcy. In order to avoid bankruptcy, the supplier reduces its borrowing from the bank (equity-to-asset ratio increases) and the trade credit factored to the customers as well.

D Proofs

Proposition 7 *The delta of the composite option whose payoff is the expectation of (24) is given by*[14]

$$\Delta_t^c = \Delta_t^{c,1} + \Delta_t^{c,2} + \Delta_t^{c,3} \tag{45}$$

where

$$\begin{aligned}
\Delta_c^1 &= (1-\alpha)(1-\tau)\big(\Phi(\bar{d}-\sigma_c\sqrt{T}) - \Phi(\underline{d}-\sigma_c\sqrt{T})\big) \\
&\quad - \frac{(1-\alpha)(1-\tau)}{\sigma_c\sqrt{T}}\big(\phi(\bar{d}-\sigma_c\sqrt{T}) - \phi(\underline{d}-\sigma_c\sqrt{T})\big) + \frac{R_D^c}{\sigma_c\sqrt{T}}\big(\phi(\bar{d}) - \phi(\underline{d})\big) \\
\Delta_c^2 &= \frac{e^{-r_f T}}{\sigma_c\sqrt{T}}\Bigg(R_{\mathcal{F}}^c\big(\phi(\hat{d}_+)\Phi(\bar{d}_-) - \Phi(\hat{d}_+)\phi(\bar{d}_-) + \Phi(\hat{d}_+)\phi(d_-) - \phi(\hat{d}_+\Phi(d_-))\big) \\
&\quad + (1-\alpha)(1-\tau)\big(\Phi(\hat{d}_-)\phi(\bar{d}_-) - \phi(\hat{d}_-)\Phi(\bar{d}_-) - \Phi(\hat{d}_-)\phi(d_-) + \phi(\hat{d}_-)\Phi(d_-)\big)\Bigg) \\
\Delta_c^3 &= \frac{R_{\mathcal{F}}^c}{\sigma_c\sqrt{T}}\phi\left(\frac{-\log\bar{\zeta}^c - \frac{1}{2}\sigma_c^2 T}{\sigma_c\sqrt{T}}\right) \\
\underline{d} &= \frac{1}{\sigma_c}\log\left(\frac{R_D^c}{(1-\alpha)(1-\tau)}\right); \quad \bar{d} = \frac{\log\zeta^c}{\sigma_c} \\
\hat{d}_+ &= \frac{-\log\hat{\zeta}^c - \frac{1}{2}\sigma_c^2 T}{\sigma_c\sqrt{T}}; \quad \hat{d}_- = \frac{\log\hat{\zeta}^c - \frac{1}{2}\sigma_c^2 T}{\sigma_c\sqrt{T}} \\
\bar{d}_- &= \frac{\log\bar{\zeta}^c + \frac{1}{2}\sigma_c^2 T}{\sigma_c\sqrt{T}}; \quad d_- = \frac{\log\zeta^c + \frac{1}{2}\sigma_c^2 T}{\sigma_c\sqrt{T}}
\end{aligned}$$

[14]We suppress the index j for ease of exposition.

Proof: Assume that the asset equation for the customers is given by

$$\log A_T^c = \log A_0^c - \frac{1}{2}\sigma_c^2 T + \sigma_c \sqrt{T} Z^c$$

The asset forms the underlying for the option whose payoff at time T is given by (24). The option is priced by taking the expectation at time 0. That is,

$$\tilde{\Pi}_c = E_0[\mathbb{1}_{A_T^c < \zeta^c} \max\{0, (1-\alpha)(1-\tau)A_T^c - R_D^c\} + \mathbb{1}_{\zeta^c < A_T^c < \bar{\zeta}^c} \hat{V}_w^c + \mathbb{1}_{\bar{\zeta}^c < A_T^c} R_{\mathcal{F}}^c] \tag{46}$$

Let the first part of the above equation (46) be defined as ,

$$\tilde{\Pi}_c^1 := E_0[\mathbb{1}_{A_T^c < \zeta^c} \max\{0, (1-\alpha)(1-\tau)A_T^c - R_D^c\}]$$

$$\begin{aligned}
\tilde{\Pi}_c^1 &= E_0[\mathbb{1}_{z<\bar{d}} \mathbb{1}_{z>\underline{d}} (1-\alpha)(1-\tau)A_T^c] - E[\mathbb{1}_{z<\bar{d}} \mathbb{1}_{z>\underline{d}} R_D^c] \\
&= (1-\alpha)(1-\tau)\frac{1}{\sqrt{2\pi}} \int_{\underline{d}}^{\bar{d}} A_0^c e^{-\frac{1}{2}\sigma_c^2 T + \sigma_c \sqrt{T} z} e^{-0.5z^2} dz \\
&= (1-\alpha)(1-\tau)A_0^c \{\Phi(\bar{d} - \sigma_c\sqrt{T}) - \Phi(\underline{d} - \sigma_c\sqrt{T})\} - R_D^c\{\Phi(\bar{d}) - \Phi(\underline{d})\}
\end{aligned}$$

where the last equality is from change of variables, and $\underline{d}, \bar{d}$ are given as

$$\begin{aligned}
\bar{d} &= \frac{\log \zeta^c + \frac{1}{2}\sigma_c^2 T - \log {A_0}^c}{\sigma_c \sqrt{T}} \\
\underline{d} &= \frac{1}{\sigma_c \sqrt{T}} \left[\log \frac{R_D^c}{(1-\alpha)(1-\tau)} - \log A_0^c + \frac{1}{2}\sigma_c^2 T \right]
\end{aligned}$$

Let the second part of the equation (46) be defined as

$$\tilde{\Pi}_c^2 := E_0[\mathbb{1}_{\zeta^c < A_T^c < \bar{\zeta}^c} \hat{V}_w^c]$$

Then, we have

$$\begin{aligned}
\tilde{\Pi}_c^2 &= E_0\Big[\mathbb{1}_{\zeta^c<A_T^c<\bar{\zeta}^c} e^{-(rf+\delta)T}\Big(R_{\mathcal{F}}^c \Phi\Big(\frac{-\log\hat{\zeta}^c - \frac{1}{2}\sigma_c^2 T + \log A_0^c}{\sigma_c\sqrt{T}}\Big) \\
&\quad + (1-\alpha)(1-\tau)\Phi\Big(\frac{\log\hat{\zeta}^c - \frac{1}{2}\sigma_c^2 T - \log A_0^c}{\sigma_c\sqrt{T}}\Big)\Big)\Big] \\
&= e^{-(rf+\delta)T} P(\zeta^c < A_T^c < \bar{\zeta}^c)\Big[R_{\mathcal{F}}^c \Phi\Big(\frac{-\log\hat{\zeta}^c - \frac{1}{2}\sigma_c^2 T + \log A_0^c}{\sigma_c\sqrt{T}}\Big) \\
&\quad + (1-\alpha)(1-\tau)\Phi\Big(\frac{\log\hat{\zeta}^c - \frac{1}{2}\sigma_c^2 T - + \log A_0^c}{\sigma_c\sqrt{T}}\Big)\Big] \\
&= e^{-(rf+\delta)T}\Big[R_{\mathcal{F}}^c \Phi\Big(\frac{-\log\hat{\zeta}^c - \frac{1}{2}\sigma_c^2 T + \log A_0^c}{\sigma_c\sqrt{T}}\Big) + (1-\alpha)(1-\tau)\Phi\Big(\frac{\log\hat{\zeta}^c - \frac{1}{2}\sigma_c^2 T - \log A_0^c}{\sigma_c\sqrt{T}}\Big)\Big] \\
&\quad * \Big\{\Phi\Big(\frac{\log\bar{\zeta}^c - \log A_0^c + \frac{1}{2}\sigma^2 T}{\sigma_c\sqrt{T}}\Big) - \Phi\Big(\frac{\log\zeta^c - \log A_0^c + \frac{1}{2}\sigma_c^2 T}{\sigma\sqrt{T}}\Big)\Big\}
\end{aligned}$$

Finally, the third part of the equation (46) is given by

$$\begin{aligned}
\hat{\Pi}_c^3 &:= E_0[\mathbb{1}_{\bar{\zeta}^c<A_T^c} R_{\mathcal{F}}^c] \\
&= R_{\mathcal{F}}^c P(\bar{\zeta}^c < A_T^c) \\
&= R_{\mathcal{F}}^c \Phi\Big(\frac{\log A_0^c - \log\bar{\zeta}^c - \frac{1}{2}\sigma_c^2 T}{\sigma_c\sqrt{T}}\Big)
\end{aligned}$$

Combining the three parts, we get the price of the required option. The delta of the option can be computed by differentiating with respect to the underlying asset A_0^c. Plugging in $A_0^c = 1$, we get the proposition 7. ∎

D.1 Proof of Theorem 3

The profit of H-customer is given by $\Pi^H = q^H(P(q) - m + \phi)$ where the downward sloping demand function $P(q)$ is given by $P(q) = \alpha - q^H - q^L$ with α denoting the demand shock. Similarly, the profit function of L-customer is given by $\Pi^L = q^H(P(q) - m)$ since she does not receive the discount ϕ. Substituting the demand function into the profit functions and equalizing the individual demands, we get the desired result.

The threshold α^* can be found by the inequality $(q^L)^2 < K$ where K is an exogenous input production cost.

D.2 Proof of Lemma 5

The proof follows from Gianetti et al 2021. Assume that the realization of demand shock $\alpha^* < \tilde{\alpha} = \alpha^* + 3\epsilon$. Then, the quantity demanded by H-customer is given by $q^H = \frac{1}{3}(\tilde{\alpha} - m) = \sqrt{K} + m$. If the maximum quantity that H-customer can purchase on discount is smaller than or equal to $\sqrt{K}$, then she can purchase $\sqrt{K}$ on discount, and the remaining m without a discount. Thus, the discount applies only for quantities corresponding to low levels of demand shock $\tilde{\alpha} < \alpha^*$. Since for low levels of demand shock, the L-customer chooses not to be active in the downstream market, there is no distortion.