Optimism, Net Worth Trap, and Asset Returns[*]

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Abstract

How does optimism about growth prospects affect the stability of the macroeconomy and its transition dynamics? We set out a tractable framework that incorporates optimism about long-run growth, and assess its impact on the amplification of boom-bust cycles. When experts become more optimistic about technological growth, trade gets facilitated, raising investment, asset prices, and leverage considerably. However, when the economy transitions into a crisis, optimism exacerbates aggregate risk, resulting in excess volatility and higher risk-premium. In some cases, the experts' expectation error stemming from their optimism generates a *net worth trap*, a situation in which the net worth of optimistic experts is trapped at very low levels indefinitely. Our model explains the excess conditional momentum in asset returns, which is empirically observed in crises, and raises the pricing power in the cross-section of asset returns significantly, beyond the conventional factors proposed by the literature.

Keywords: Optimism, Net Worth Trap, Conditional Momentum

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1 Introduction

There has been a surge in the literature on macroeconomic models with financial frictions post 2008 financial crisis, aiming to explain crises and high risk premium levels on financial assets. However, the majority of these models neglect cognitive beliefs about the macroeconomic state, with only a few notable exceptions (e.g., Krishnamurthy and Li (2020), Maxted (2023), Khorrami and Mendo (2023), Camous and Van der Ghote (2023) etc.). Our paper aims to bridge this gap by investigating how cognitive beliefs of market participants about the long-run macroeconomic state of economy influence financial stability and affect aggregate asset returns.

This paper contributes in two key ways. First, we introduce a tractable model with two types of agents who differ in terms of their beliefs about macroeconomic growth. One type, who is more productive in capital utilization, holds optimistic beliefs about the economy's long-run growth rate, which contrasts with the other type who is less productive but has rational beliefs.[1] The optimism of the productive agents results in expectation errors in their capital returns, leading to financial instability at the aggregate level. Moreover, beyond a certain threshold level of optimism, these forecast errors generate a 'net worth trap', a phenomenon that concentrates capital exclusively in the hands of less productive agents and perpetuates long-term inefficiencies. In addition, optimism negatively impacts the welfare of rational agents, primarily by generating a net worth trap where capital prices and the economy's growth rates are depressed for a long period. This effect on capital prices from the optimism generates and matches empirically documented return predictability patters, constituting the second contribution of our paper. The interaction between optimism and the net worth of productive agents in explaining the risk premium inherently suggests a two-factor asset pricing model. To validate its pricing effectiveness, we construct an optimism factor using data from The Survey of Professional Forecasters. The inclusion of optimism in the factor pricing model effectively prices the cross-section of asset returns, showcasing improved performance compared to the traditional factors typically utilized in empirical asset pricing studies.

We purposefully build a model with a strong emphasis on financial frictions and risk premia following He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), Gertler et al. (2020), etc., to study implications on financial and macroeconomic stability as well as

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[1]This assumption aligns with the literature that portrays entrepreneurs as skilled and optimistic. See e.g., Coval and Thakor (2005). Following the literature, we call more and less productive agents 'experts' and 'households', respectively.

on asset returns. In these papers, the net worth of the leveraged agents is a natural measure of financial stability. The main addition to this framework is a dogmatically optimistic belief of more productive users of capital, reflecting the empirical evidence found in e.g., Puri and Robinson (2007). The first main insight that comes out of the model is that optimism exacerbates financial instability, resonating Camous and Van der Ghote (2023).[2] The mechanisms that create the instability are that optimists expect a higher growth rate of economy than the true growth rate, and this expectation error influences their portfolio choice over risky capital. As they operate with leverage in equilibrium, a negative shock in the economy depresses their net worth due to a loss in the capital price arising from fire-selling the risky capital to less productive agents. Subsequent negative shocks create two effects: (i) a positive effect that aids the optimists in rebuilding their net worth through high risk premium, and (ii) a negative effect stemming from the expectation error that counteracts the net worth rebuilding process. The second effect arises because the optimists take their portfolio decisions based on the perceived growth rate which is higher than the true growth rate of economy. For lower levels of optimism, the first effect dominates, leading the economy to allocate all capital to the productive optimists in the future. However, beyond a threshold level of optimism, the second effect dominates wiping out the optimists wealth and driving the economy to allocate all capital to less productive agents. We call this phenomenon a "net worth trap", in which the optimists remain unable to rebuild their wealth. This trap does not feature in a rational expectations benchmark model with no expectation errors. While models with financial frictions and belief distortions such as Krishnamurthy and Li (2020) and Maxted (2023) feature a fat tailed stationary distribution, we offer analytical characterization of an extreme case of the instability where the stationary distribution endogenously becomes a dirac measure at the most inefficient region.[3]

Beyond increasing the financial instability, macro-level optimism generates a welfare loss for the rational agents in the economy. We perform a decomposition of the welfare to understand different channels contributing to welfare loss. The decomposition leads to four channels: (i) wealth effect, (ii) investment effect, (iii) capital effect, and (iv) misallocation effect. The wealth effect captures the wealth share of rational agents, and the investment

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[2]Camous and Van der Ghote (2023) builds a model based on diagnostic expectations where risk-neutral agents face binding leverage constraints and the financiers of capital do not consume. In our model, agents are risk-averse with preferences over consumption, and optimists face a skin-in-the game constraint.

[3]In fact, our model does not have any additional assumptions such as portfolio and collateral constraints, or large exit that is shown to be required to generate a net worth trap. See Gopalakrishna (2022) who introduces fluctuating productivities and state-dependent exits of experts to generate a similar net worth trap.

effect captures the welfare attributed to capital investment. These two positive effects raises welfare as the optimism of productive agents increase. First, a larger optimism increases financial instability implying a larger (smaller) wealth share of rational (optimistic) agents. Second, higher optimism depresses the investment rate, and mechanically increases the welfare of rational agents since they have more to consume. The third effect is due to the growth rate of aggregate capital. With higher optimism levels, the aggregate capital grows slowly due to a poor investment rate, decreasing the welfare. Finally, since more capital is allocated to less productive agents as optimism increases, the misallocation effect dampens their welfare. The last two effects turn out to dominate and increase the utility loss for the rational households.

In addition to explaining the welfare loss of rational agents, fluctuations in capital price play a key role in explaining empirical asset returns, both in the time-series and the cross-section. In the time-series, the model matches the asset return predictability patterns documented empirically. The mechanism is as follows: in comparison to a rational expectation benchmark model, our model results in a large drop in the net worth of optimists following a sequence of adverse shocks, subsequently leading to an increase in the risk premium. For moderate levels of optimism, the optimists' expectation error is dominated by the risk premium earned on risky capital, facilitating the rebuilding of their net worth. This recapitalization induces a momentum in the asset price, with the momentum effect intensifying in optimism level.

Another implication of our model is that optimism, though is assumed to be static, generates time-varying risk premium through fire-selling of risky capital to less efficient agents. To validate this model prediction, we empirically construct a factor-based asset pricing model, using the quarter-on-quarter GDP growth rate from The Survey of Professional Forecasters (SPF) as a proxy for optimism. We show that our factor model, incorporating both optimism and the net worth factor (the latter being proxied by an intermediary capital based factor following He et al. (2017)), improves the pricing power of a broad spectrum of test assets above and beyond the capabilities of traditional factors. The optimism factor carries a positive price of risk, both unconditionally and conditional on the exposure to net worth factor.

The elicitation of objectively accurate beliefs about the underlying technological process (i.e., rational expectations) is out of the scope of this paper. Optimists in our model have static beliefs about their expected technological growth rate, and are dogmatic in their own views. In this formulation, optimism is persistent and optimists never learn from the

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data. This setting is in accordance with the literature where variation about individual beliefs about expected returns are due to individual fixed effects (e.g., Giglio et al. (2019)). In contrast to the models of e.g., Harrison and Kreps (1978), Scheinkman and Xiong (2003), and Geanakoplos (2010), we consider risk-averse, not risk-neutral agents. On the methodological side, we normalize the economy by technology so that our economy becomes stationary, and rely on the Kolmogorov forward equation (KFE) in characterizing the ergodic distribution of our state variable, in order to illustrate the long term dynamics and especially, the net worth trap.

Related literature   Our paper relates to a large literature on macro-finance models with financial frictions playing a central role in amplifying shocks (e.g., He and Krishnamurthy (2013), Gertler et al. (2020) Brunnermeier and Sannikov (2014)). Recent works in this literature have turned to quantitative models that explain macroeconomic and asset pricing moments either using a rational expectations model, or using a belief based model.[4] We follow this line of literature but focus on the general equilibrium interactions between the endogenous risk and macroeconomic variables in the presence of optimism about the long-run growth. Specifically, we build a tractable model with optimism that generates a net worth trap and conditional momentum in asset returns consistent with the empirical evidence. While we focus on optimism as a major behavioral factor, it relates to the literature that focuses on both optimism and pessimism, or belief heterogeneity in general (see, for example, Harrison and Kreps (1978), Simsek (2013), and and Caballero and Simsek (2020)).[5]

We also relate to a long literature on deviations from the rational expectations, e.g., Detemple and Murthy (1997), Basak and Croitoru (2000), Basak (2000), Basak and Croitoru (2006), Gallmeyer and Hollifield (2008), Chabakauri (2015), and Dong et al. (2022).[6] The difference is that we focus on the implications on financial stability and the interaction be-

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[4]Maxted (2023) builds a macro-finance model with diagnostic expectations, and Krishnamurthy and Li (2020) builds a model where agents update their beliefs about tail risk rationally. In a recent work, Gopalakrishna (2022) introduces stochastic productivity and state-dependent exits of experts into a canonical rational expectations model to generate amplified (in volatility and risk-premium) but 'slow-moving' financial crises.

[5]Harrison and Kreps (1978) assume that agents agree to disagree about their beliefs, and asset prices can exceed their fundamental values in that case. Simsek (2013) studies cases in which optimists borrow from pessimists using loans collateralized by the asset that optimists purchase. As pessimists attach lower values to the collateral, this collateral arrangement puts the endogenous borrowing constraint on optimists, affecting their leverage choices and asset prices.

[6]In Dong et al. (2022), optimistic investors buy risky assets with leverage provided by pessimists, pushing up asset prices like in our model. Higher asset prices relax the financial frictions imposed on high productivity firms, mitigating degrees of misallocation and raising aggregate output.

tween macroeconomy and asset prices. In this regard, it is similar to Maxted (2023) and Camous and Van der Ghote (2023). Camous and Van der Ghote (2023) introduces diagnostic expectations into a canonical macro-finance model with binding leverage constraints and shows that diagnostic expectations exacerbate financial instability. On the contrary, the diagnostic expectation decreases financial instability compared to a rational expectation benchmark in Maxted (2023). Our paper's financial stability results are closer to the former, and in addition, characterizes an extreme case of instability featuring a net worth trap where the stationary distribution has a sharp peak around the zero net worth of experts.

Finally, this paper relates to the empirical literature focusing on heterogeneous beliefs across different groups of market agents. For example, Welch (2000) finds that there is a large degree of heterogeneity in forecasted risk premium levels even across financial economists, while Beutel and Weber (2022) point out individuals are heterogeneous both at the information acquisition and the processing stages, thereby forming their own beliefs and choosing portfolios based on those beliefs.[7] Our results are based on the *perceived* equity premium levels across two groups (i.e., optimists and households), providing novel implication about the interaction between optimistic belief and the crisis dynamics in general equilibrium.

Outline   The remainder of the paper is organised as follows: Section 2 sets out the basic framework. In particular, Section 2.4 characterizes the equilibrium in an analytic way and Section 2.5 provides simulation results and discusses our model's implications. Section 3 provides our quantitative analysis in regards to the conditional excess momentum in asset returns and cross-sectional asset pricing implications. Section 4 concludes. Appendix A provides additional figures and robustness checks, and Appendix B provides omitted proofs and derivations.

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[7]The literature also points out that different groups in the economy (e.g., households, firms, professional forecasters, etc) form different expectations not just about risk premium but also about macroeconomic variables: see e.g., Coibion et al. (2020), Candia et al. (2021), and Weber et al. (2022). As risk premium depends on the business cycle (e.g., Cooper and Priestley (2009)), we can expect that the forecasted risk-premium levels across groups would differ. For equity premium, Rapach et al. (2012) uncover that despite the failure of *individual* out of sample forecasts to outperform the historical average, *combinations* of individual forecasts deliver significant out-of-sample gains relative to the historical average on a consistent basis over time.

2 The Model

We develop a continuous-time framework with two types of agents: optimists and households, based on which we study how optimism about technological growth affect leverage choices, asset prices and the endogenous financial volatility, where the endogenous volatility itself affects the optimists' degree of optimism in asset returns. Our setting is analytical yet tractable, and incorporates exogenous technological growth and heterogeneous beliefs in a general equilibrium sense. Our model is built on e.g., Basak (2000) and Brunnermeier and Sannikov (2014).

2.1 Model Setup

We begin with the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ which is endowed with a standard Brownian motion Z_t. We assume that $Z_0 = 0$, almost surely. All economic activity will be assumed to take place in the horizon $[0, \infty)$. Let

$$\mathcal{F}^Z(t) \triangleq \sigma\{Z_s;\ 0 \leq s \leq t\}, \forall t \in [0, \infty)$$

be the filtration generated by $Z(\dot{)}$ and let $\mathcal{N}$ denote the $\mathcal{P}$- null subsets of $\mathcal{F}^Z(\infty)$. We shall use the augmented filtration as follows:

$$\mathcal{F}(t) \triangleq \sigma\{\mathcal{F}^Z(t) \cup \mathcal{N}\}, \forall t \in [0, \infty).$$

One should interpret the $\sigma-$algebra $\mathcal{F}(t)$ as the information available to agents at time t in a complete information setting, in the sense that if $\omega \in \Omega$ is the true state of nature and if $A \in \mathcal{F}(t)$, then all agents will know whether $\omega \in A$.

We consider an economy with two types of agents, 'optimists' and 'households'. Both types of agents can own capital, but the former are able to use capital in a more productive way.[8]

2.1.1 Technology

The aggregate amount of capital in the economy is denoted by K_t and capital owned by an individual agent i by k_t^i, where $t \in [0, \infty)$ indicates time. The physical capital k_t^O held by

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[8]Our assumption that more productive intermediaries are more optimistic than the rational households is from the literature including Coval and Thakor (2005) and Chabakauri (2015). Still our analysis can embed the opposite case in which more productive intermediaries are pessimistic about growth.

optimists produces output at rate:

$$y_t^O \triangleq \gamma_t^O k_t^O, \quad \forall t \in [0, \infty) \tag{1}$$

per unit of time, where γ_t^O is an exogenous productivity parameter, which evolves according to:[9]

$$\frac{d\gamma_t^O}{\gamma_t^O} \triangleq \alpha dt + \sigma dZ_t, \quad \forall t \in [0, \infty), \tag{2}$$

where dZ_t is the aggregate standard Brownian motion defined above. The output is modeled as a numeraire, and therefore, its price is normalized to one. Capital owned by individual optimists, with state space $F^k \subseteq \mathbb{R}$, satisfies the following Ito's process:

$$\frac{dk_t^O}{k_t^O} \triangleq \left(\Lambda^O(\iota_t^O) - \delta^O \right) dt, \quad \forall t \in [0, \infty), \tag{3}$$

where ι_t^O is the portion of the generated output (i.e., $y_t^O = \gamma_t^O k_t^O$) used in creating new capital (i.e., the investment made during an infinitesimal period $(t, t + dt)$ is $\iota_t^O \gamma_t^O_t k_t^O dt$). In (3), function $\Lambda^O(\cdot)$, which satisfies $\Lambda^O(0) = 0$, $\Lambda^{O\prime}(0) = 1$, $\Lambda^{O\prime}(\cdot) > 0$, and $\Lambda^{O\prime\prime}(\cdot) < 0$, represents a standard investment technology with adjustment costs. In cases where there is no investment, the capital managed by optimists depreciates at rate δ^O. The concavity of $\Lambda^O(\cdot)$ represents some investment formation friction, which is interpreted as adjustment costs of converting the output to the new capital and vice versa.[10]

Households are less productive. The capital managed by the households, with corresponding state space $F^k \subseteq \mathbb{R}$, produces the following output:

$$y_t^H \triangleq \gamma_t^H k_t^H, \quad \forall t \in [0, \infty), \tag{4}$$

with γ_t^H is given by $\gamma_t^H = l \cdot \gamma_t^O \leq \gamma^O$, where $l \leq 1$, and evolves according to:

$$\frac{d\gamma_t^H}{\gamma_t^H} = \frac{d\gamma_t^O}{\gamma_t^O}, \quad \forall t \in [0, \infty). \tag{5}$$

In other words, optimists have a proportionally higher productivity than households, where the proportionality is given by $l \leq 1$, and productivity of two groups of agents grows at the

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[9]Therefore, unlike Brunnermeier and Sannikov (2014), we have the stochastic growth which we model as exogenous (i.e., α and σ in (2) are exogenous). Later we normalize our economy by γ_t^O to make it stationary.

[10]In contrast to the literature (e.g., Brunnermeier and Sannikov (2014) and Gopalakrishna (2022)), we abstract from the capital risk usually assumed. Its inclusion does not change our results qualitatively.

same rate at every instant. Finally, the capital owned by households, which we denote by k_t^H follows:

$$\frac{dk_t^H}{k_t^H} \triangleq \left(\Lambda^H(\iota_t^H) - \delta^H \right) dt, \quad \forall t \in [0, \infty), \tag{6}$$

where in the same way as above, ι_t^H is a capital-owning pessimist's investment rate per unit of output. The state space F^k satisfies the same conditions as for optimists and $\delta^H \geq 0$ is the depreciation rate when the capital is managed by households. We assume that $\Lambda^H(\iota) = l \cdot \Lambda^O(\iota)$ for $\forall \iota$ with $l < 1$ for simplicity.[11]

2.1.2 Preferences

Optimists and households have preferences that are generally characterized by the instantaneous utility function $u^i(c_t^i) : \mathbb{R}_+ \to \mathbb{R}$, where $i \in \{O, H\}$, and each group i has a constant discount factor ρ^i. The consumption space defined above must also be square-integrable:

$$\int_0^\infty \left| c_t^i \right|^2 dt < \infty.$$

With $c^i \equiv \{c_t^i\}_{t=0}^\infty$, each agent of type $i \in \{O, H\}$ maximizes her expected lifetime utility given by:

$$U(c^i) \triangleq \int_0^\infty e^{-\rho^i t} u^i(c_t^i) dt, \quad \forall t \in [0, \infty). \tag{7}$$

The utility function obeys the standard assumption that $u^i(c_t^i)$ is continuously differentiable, increasing, and concave for $i \in \{O, H\}$.

2.1.3 Market for Capital

Both optimists and households have the opportunity to trade the physical capital in a competitive market. We denote the equilibrium market price of capital in terms of the output by p_t with corresponding state space $F^p \subseteq \mathbb{R}$, and assume it satisfies the following endogenous Ito's process:

$$\frac{dp_t}{p_t} \triangleq \mu_t^p dt + \sigma_t^p dZ_t, \tag{8}$$

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[11]Therefore, we effectively assume that both (i) households' productivity in turning capital into output; and (ii) their productivity in turning output into capital are both lower than of optimists with the same proportionality $l \leq 1$. This is for tractability and even if we assume different proportionality for the two productivity measures, most results in this paper do not change qualitatively. Thus, we interchangeably call optimists 'experts' as in Gopalakrishna (2022).

where μ_t^p and σ_t^p are drift and volatility of the capital price process (8), respectively. Based on the definition, capital k_t^O costs $p_t k_t^O$ for optimists. Note that, in equilibrium, p_t, μ_t^p and σ_t^p are determined endogenously.

2.1.4 Return to Capital

A straightforward application of Ito's lemma in (3) and (8) reveals that when an optimists hold k_t^O units of capital at price p_t, the total value of this capital (i.e., $p_t k_t^O$) evolves according to:

$$\frac{d(p_t k_t^O)}{p_t k_t^O} = \left(\Lambda^O(\iota_t) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t. \tag{9}$$

Hence, the total return that experts earn from capital (per unit of wealth invested) is given by:

$$dr_t^{Ok} = \frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t. \tag{10}$$

Similarly, a household earns the return of

$$dr_t^{Hk} = \frac{\gamma_t^H - \iota_t^H \gamma_t^H}{p_t} dt + \left(\Lambda^H(\iota_t^H) - \delta^H + \mu_t^p\right) dt + \sigma_t^p dZ_t. \tag{11}$$

2.1.5 Optimism

The households are rational, i.e., they know that their productivity γ_t^H follows the process in (2) and (5). Optimists, in contrast, observe their productivity γ_t^O at any instant, but have incomplete information on its exact dynamics. With (2), we know that γ_t^O follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha dt + \sigma dZ_t, \quad \forall t \in [0, \infty). \tag{12}$$

Optimists are assumed to believe that their productivity γ_t^O follows instead

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha^O dt + \sigma dZ_t^O, \quad \forall t \in [0, \infty), \tag{13}$$

where α^O is possibly different from the true α and Z_t^O is their own Brownian motion. We assume that $\alpha^O > \alpha$, with which the optimists believe in their productivity's higher growth rate than its true growth in (2). Optimists are *dogmatic* and do not learn from the realized data, in a similar manner to Yan (2008) and Chabakauri (2015).

From (12), and (13), we obtain the following consistency condition for optimists:

$$Z_t^O = Z_t - \frac{\alpha^O - \alpha}{\sigma} t. \tag{14}$$

In other words, optimists regard Z_t^O, not Z_t, as the Brownian motion driving the business cycle, while Z_t^O is in fact not a Brownian motion under the rational expectations. With the assumption $\alpha^O \geq \alpha$,[12] we call (14) the optimism process. From (10) and (14), the optimists believe that the total return that they earn from capital holding (per unit of wealth invested) would be given by

$$dr_t^{Ok} = \frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^O - \alpha}{\sigma} \sigma_t^p \right) dt + \sigma_t^p dZ_t^O. \tag{15}$$

Two points regarding equation (15) are important to note: (i) with $\alpha^O \geq \alpha$, optimists believe that the expected capital gain they earn when investing in physical capital is higher than implied under the rational expectations; (ii) the degree of optimism (i.e., $\frac{\alpha^O - \alpha}{\sigma} \sigma_t^p$ in terms of the 'expected' return becomes proportional to the endogenous capital price volatility σ_t^p: thereby, a higher endogenous risk σ_t^p mechanically raises the degree of optimism in asset returns.[13]

2.2 Consumption-Portfolio Problems

Optimists Optimists can invest in the physical capital, or the risk free asset, which is zero net supplied. The non-monetary net worth w_t^O of each optimist who invests fraction x_t^O of her wealth in capital and consumes with the rate c_t^O evolves according to:

$$dw_t^O = x_t^O w_t^O dr_t^{Ok} + (1 - x_t^O) r_t w_t^O dt - c_t^O dt, \tag{16}$$

where r_t is the risk-free interest rate prevailing in the economy. Note that r_t is an equilibrium object to be determined endogenously. x_t^O represents the share of wealth the optimists invest in capital. Later, it will turn out that in most cases optimists use greater-than-0 leverage (i.e., $x_t^O > 1$). In most cases, we call x_t^O 'leverage' or 'leverage multiple' of optimists.

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[12]The case of $\alpha^O < \alpha$ would correspond to the case in which experts are pessimists. Our analytical results do not rely upon the relative sizes of α^O though.

[13]Compared with Brunnermeier and Sannikov (2014) where more productive experts have higher dividend yields given the same investment amount, our framework features an additional channel where they become more optimistic about future returns through higher capital gain in the case of $\alpha^O \geq \alpha$.

Formally, each optimist solves

$$\max_{x_t^O \geq 0, c_{t,}^O \geq 0} \mathbb{E}_0^O \left[\int_0^\infty e^{-\rho t} u^O \left(c_t^O \right) dt \right], \tag{17}$$

subject to the solvency constraint $w_t^O \geq 0$ and the dynamic budget constraint (16). In optimization (17), the expectation operator $\mathbb{E}_0^O$ means that optimists believe dZ_t^O, not dZ_t, is the true Brownian motion. Therefore in characterizing (16), they use (15) for the capital return process dr_t^{Ok} instead of (10) with the true dZ_t. Note that we assume away short-sale for capital, thereby $x_t^O \geq 0$.

Households   In a similar way to optimists' problem in (17), the non-monetary net worth w_t^H of households who invest fraction x_t^H of their wealth in capital and consume with the rate c_t^H would follow

$$dw_t^H = x_t^H w_t^H dr_t^{Hk} + w_t^H (1 - x_t^H) r_t dt - c_t^H dt. \tag{18}$$

Formally, each household solves

$$\max_{x_t^H \geq 0, c_{t,}^H \geq 0} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} u \left(c_t^H \right) dt \right], \tag{19}$$

subject to the solvency constraint $w_t^H \geq 0$ and the dynamic budget constraints (18). Unlike optimists, in optimization (19), the expectation operator $\mathbb{E}_0$ means that households have the rational expectations: they believe dZ_t is the true Brownian motion. Therefore in characterizing (18), they use (11) for the capital return process dr_t^{Hk} with the true dZ_t. Note that we assume away short-sale for capital, thereby $x_t^H \geq 0$.

2.3   Equilibrium and Market Clearing

Intuitively, an equilibrium with full information is characterized by a map from shock histories $\{Z_S, s \in [0,t]\}$, to the prices p_t and asset allocations such that, given prices, agents maximize their expected utilities[14] and markets clear.[15] To define our equilibrium more formally, we denote the set of optimists by an interval $I = [0,1]$ and index any optimist (i.e.,

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[14]The optimists solve optimization (17) subject to their dynamic budget constraint (16) and the solvency $w_t^O \geq 0$ while the households solve optimization (19) subject to their dynamic budget constraint (18) and the solvency $w_t^P \geq 0$.

[15]The physical capital, output, and debt markets must clear in equilibrium.

expert) by $i \in I = [0, 1]$. Similarly, we denote a set of the households by $J = (1, 2]$ with index $j \in J = (1, 2]$. Since optimists deviate from the rational expectations with their own perceived shocks Z_t^O, they have their own expectation operator $\mathbb{E}_0^O$ as shown in (17). Note that in equilibrium, every agent in the same group (i.e., optimists or households) chooses the same consumption and portfolio decisions.

We now state the market clearing conditions and formally define the equilibrium. The three markets that must clear in equilibrium at any given instant are the physical capital, consumption good, and risk-free debt markets.

Capital Market The total amount of the capital demanded by optimists and households is equal to the aggregate supply of capital in the economy: i.e.,

$$\int_0^1 k_t^O \, di + \int_1^2 k_t^H \, dj = K_t, \ \ \forall t \in [0, \infty), \tag{20}$$

where K_t is the total supply of capital. The total supply of capital in the model is not fixed as both optimists and households invest in the new capital through their investments. The following (21) describes the evolution of the total supply of capital:

$$dK_t \triangleq \left(\int_0^1 \left(\Lambda^O \left(\iota_t^O \right) - \delta^O \right) k_t^O \, di + \int_1^2 \left(\Lambda^H \left(\iota_t^H \right) - \delta^H \right) \underline{k}_t^H \, dj \right) dt, \ \ \forall t \in [0, \infty). \tag{21}$$

Good Market Whatever is produced and not invested, has to be consumed. That is,[16]

$$\int_0^1 k_t^O \left(\gamma_t^O - \iota_t^O \gamma_t^O \right) di + \int_1^2 k_t^H \left(\gamma_t^H - \iota_t^H \gamma_t^H \right) dj = \int_0^1 c_t^O \, di + \int_1^2 c_t^H \, dj, \ \ \forall t \in [0, \infty). \tag{22}$$

Debt Market The debt market clearing condition implies that the value of the debt that optimists receive should be equal to the value of loans that the households extend, namely,

$$\int_0^1 \left(w_t^O - p_t k_t^O \right) di + \int_1^2 \left(w_t^H - p_t k_t^H \right) dj = 0. \tag{23}$$

By defining all three markets, we are in a position to define the economy's equilibrium.

Definition 1 *The equilibrium consists of the stochastic processes of (i) the price of capital*

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[16]In equation (22), we use the fact that the investment rate at time t is given by $\iota_t^i \gamma_t^i k_t^i$ for $i \in [0, 2]$.

p_t, (ii) interest rate r_t, (iii) investment rate and consumption, i.e. $\left\{\left(k_t^O, \iota_t^O, c_t^O\right), t \geq 0\right\}$ for optimists, and $\left\{\left(k_t^H, \iota_t^H, c_t^H\right), t \geq 0\right\}$ for households, which should satisfy the following three conditions:

1. Given their perceived Brownian motion Z_t^O, the optimists $O \in [0,1]$ solve optimization (17) subject to their dynamic budget constraint (16) and the solvency $w_t^O \geq 0$. Households $H \in (1,2]$ solve optimization (19) subject to their dynamic budget constraint (18) and the solvency $w_t^j \geq 0$, under the rational expectations.

2. The capital (i.e., (20) and (21)), consumption (i.e., (22)), and debt (i.e., (23)) markets clear.

3. The consistency condition (14) between Z_t^O and Z_t holds.

2.4 Equilibrium Characterization

In this section, we discuss how to find the equilibrium price p_t with its endogenous drift μ_t^p and volatility σ_t^p, the optimists and households' consumption and portfolio decisions, the risk-free interest rates r_t, given the history of perceived shock processes $\left\{Z_s^O, Z_s, 0 \leq s\right\}$. We first start with some definitions.

Definition 2 *The aggregate wealth of both optimists and households is given by summing up their individual wealth[17] respectively, that is,*

$$W_t^O = \int_0^1 w_t^O di, \ \ W_t^H = \int_1^2 w_t^H dj, \quad \forall t \in [0, \infty).$$

Observe that the capital market clearing condition (20) and the debt market clearing condition (23) become[18]

$$W_t^O + W_t^H = p_t K_t \tag{24}$$

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[17]Each optimist chooses the same consumption and portfolio decision, and thus has the same wealth w_t^O. Likewise, each household owns the same wealth level w_t^H in equilibrium.

[18]Equation (24) is equivalent to

$$\int_0^1 x_t^O w_t^O di + \int_1^2 x_t^H w_t^H dj = p_t K_t, \quad \forall t \in [0, \infty).$$

where K_t is the supply of aggregate capital that follows the process (21). Finally, the good market equilibrium condition (22) can be written as

$$\int_0^1 \frac{x_t^O w_t^O}{p_t} \left(\gamma_t^O - \iota_t^O \gamma_t^O \right) di + \int_1^2 \frac{x_t^H w_t^H}{p_t} \left(\gamma_t^H - \iota_t^H \gamma_t^H \right) dj = \int_0^1 c_t^O di + \int_1^2 c_t^H dj. \quad (25)$$

We now characterize the equilibrium based on the economy's state variable: the wealth share of optimists. By representing each variable in terms of optimists' wealth share which is bounded between 0 and 1, we characterize the equilibrium price and quantity variables in a similar manner to the literature (e.g., He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014)).

The proportion of wealth that optimists possesses which we denote by η_t is given by,

$$\eta_t = \frac{W_t^O}{W_t^O + W_t^H} \underset{(24)}{=} \frac{W_t^O}{p_t K_t}. \quad (26)$$

which we postulate follows the process:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta(\eta_t)dt + \sigma_t^\eta(\eta_t)dZ_t. \quad (27)$$

The economy's dynamics is driven by η_t in our Markov equilibrium as in the literature (e.g., He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014)). We have

$$x_t^O \leq \frac{1}{\eta_t}, \quad (28)$$

which translates to the fact that the maximum 'leverage' that optimists can obtain is bounded above by $\frac{1}{\eta_t}$. We identify two regions: the first one is when (28) binds and the second one is when leverage is strictly less than $\frac{1}{\eta_t}$, i.e., when (28) does not bind. We call the first region '*normal*' and the second region by '*crisis*'. In other words, in the *normal* region, all the physical capital will be owned by optimists, while in the *crisis* regime, some of the capital must be purchased by households.

Internal Investment Hereafter, we define the investment functions $\Lambda^O(\cdot)$ that optimists use as follows:

$$\Lambda^O(\iota_t^O) = \frac{1}{k} \left(\sqrt{1 + 2k\iota_t^O} - 1 \right), \quad \forall t \in [0, \infty), \quad (29)$$

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which satisfies all the standard assumptions in Section 2.1.1. We do not allow disinvestment, thus $i_t^O \geq 0$. Note that we use the mathematical form in (29) of $\Lambda^O(i_t^i)$ for simplicity and acknowledge that our results do not change qualitatively even if we use different forms for $\Lambda^O(\cdot)$ that satisfy conditions in Section 2.1.1. Similarly, we define the internal investment function $\Lambda^H(i_t^H)$ of households as

$$\Lambda^H(\iota_t^H) = l \cdot \Lambda^O(\iota_t^H), \ \ \forall \iota_t^H. \tag{30}$$

From now on, we express our equilibrium with the following normalized asset price:

Definition 3 *The normalized asset price q_t is defined as $q_t \equiv \dfrac{p_t}{\gamma_t^O}$.*

The normalized asset price q_t can be interpreted also as the price-earnings ratio of physical capital for optimists. It turns out we can characterize the 'stationary' equilibrium when we write our model in terms of q_t instead of p_t, as we have exogenous growth.

2.4.1 Solving the Consumption-Portfolio Problems

An optimist solves the optimization (17) subject to her wealth dynamics (16) and the solvency $w_t^O \geq 0$. Likewise, each household optimizes her lifetime utility (19) subject to the evolution of his wealth (18) and the solvency $w_t^H \geq 0$. We focus on the case where all the optimists and households have the same logarithmic utility function, i.e., $u^O(c_t^O) = \log c_t^O$, $u^H(c_t^H) = \log c_t^H$ for mathematical tractability.

The Proposition 1 solves the respective optimal choices of optimists and households for consumption c_t^{O*}, c_t^{H*}, leverage x_t^O, x_t^H, and investment ι_t^{O*}, ι_t^{H*}, and the equilibrium risk free interest rate r_t^*. $x_t^H \geq 0$ must hold since the short-sale of capital is not allowed.

Proposition 1 *Assume optimists and households have the logarithmic utility, i.e., $u^O(c_t^O) = \log c_t^O$, and $u^H(c_t^H) = \log c_t^H$. Then,*

1. *The optimal consumption c_t^{O*} is given by $c_t^{O*} = \rho w_t^O$. The household's consumption c_t^{H*} is given by $c_t^{H*} = \rho w_t^H$.*

2. *The equilibrium interest rate r_t^* is given by:*

$$r_t^* = \left(\frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p \right) - x_t^O \left(\sigma_t^p\right)^2,$$

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where x_t^O is each optimist's optimal portfolio choice (i.e., leverage multiple) as defined in (16). Given this r_t^*, the optimal portfolio choice of households x_t^H as defined in (18) is given by:[19]

$$x_t^H = \max \left\{ \frac{\left(\dfrac{\gamma_t^H - \iota_t^H \gamma_t^H}{p_t} + \Lambda^H(\iota_t^H) - \delta^H + \mu_t^p \right) - r_t^*}{\left(\sigma_t^p\right)^2}, 0 \right\}.$$

3. *The optimal investment rates ι_t^{O*} and ι_t^{H*} are given by:*

$$i_t^{O*}(q_t) = i_t^{H*}(q_t) = \frac{q_t^2 - 1}{2k}.$$

Note that given i_t^O, μ_t^p, σ_t^p, and x_t, the equilibrium interest rate r_t^* increases due to the optimism measure, i.e., $\alpha^O - \alpha$. If optimists believe that the expected capital gain they earn when investing in capital is higher, they will try to get more loans from households, raising the equilibrium interest rate r_t^*.

Finally, in order to conclude with the characterization of equilibrium, we need to derive the evolution of the state variable η_t. First, we define the fraction of physical capital held by optimists and pessimists by

$$\psi_t \equiv \frac{k_t^O}{K_t}, \quad 1 - \psi_t = \frac{k_t^P}{K_t}, \tag{31}$$

where the aggregate capital K_t follows the process in (21). Then, the leverage multiples of optimists and households, i.e., x_t^O and x_t^H, respectively, can be characterized from (26) and (31) as:

$$x_t = \frac{\psi_t}{\eta_t} \qquad \underline{x}_t = \frac{1 - \psi_t}{1 - \eta_t}. \tag{32}$$

2.4.2 The Aggregate Dynamics

Based on the above definitions, the evolution of the proportion of wealth held by optimists can be characterized in the following Proposition 2. Note that in Proposition 2, we express the aggregate dynamics in the true Brownian motion dZ_t.

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[19]Therefore, if the households' *perceived* risk-premium levels are below 0, they only invest in risk-free loans issued by optimists.

Proposition 2 *The evolution of optimists' wealth relative to the entire economy is given by*

$$\frac{d\eta_t}{\eta_t} = \mu^\eta(\eta_t)dt + \sigma^\eta(\eta_t)dZ_t \tag{33}$$

where

$$\mu^\eta(\eta_t) = \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p + \frac{1 - \iota_t^O}{q_t} + (1 - \psi_t)\left(\delta^H - \delta^O\right) + (1-l)\left(1 - \psi_t\right)\Lambda^O(i_t^O) - \rho,$$

and

$$\sigma^\eta(\eta_t) = \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p.$$

Now by knowing the drift μ_t^η and the volatility σ_t^η of our state variable η, we calculate the the drift μ_t^p and the volatility σ_t^p of the price of capital p_t.

Proposition 3 *When q_t is a function of state variable η_t with $q_t = q(\eta_t)$, the drift μ_t^p of the price of capital p_t is given by*

$$\mu_t^p = \alpha + \frac{q'(\eta_t)}{q_t}\mu^\eta(\eta_t)\eta_t + \frac{1}{2}\sigma^\eta(\eta_t)\eta_t\frac{q''(\eta_t)}{q(\eta_t)} + \sigma\sigma^\eta(\eta_t)\eta_t\frac{q'(\eta_t)}{q(\eta_t)}, \tag{34}$$

where $\sigma_t^\eta(\eta_t)$ is given by Proposition 2, and the volatility of the price of capital, σ_t^p, is given by

$$\sigma_t^p = \frac{\sigma}{1 - (x_t\eta_t - \eta_t)\dfrac{q'(\eta_t)}{q(\eta_t)}}. \tag{35}$$

We can observe from (33) that optimism (i.e., $\alpha^O > \alpha$) given $\psi_t \geq \eta_t$ (which holds in the efficient region[20]) lowers the drift $\mu^\eta(\eta_t)$. It implies that the optimism hinders the process of optimists' wealth recapitalization and tends to lower their wealth share in the long run, potentially generating a 'net worth trap' when the level of optimism, i.e., $\alpha^O - \alpha$, is high enough. This is caused by the optimists' expectation errors (i.e., wrong judgment about the capital return), and our main focus in Section 2.5.

We derive a closed formed solution for leverage multiple and a first-order differential equation for the price of capital in Appendix B.1.

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[20]In the efficient region, all the capital is held by optimists, thereby $\psi_t = 1$.

2.5 Model Analysis

Now, we solve our model numerically under the following calibration. The parameters with $\alpha^O = 0.13$ are chosen to target a probability of crisis of 5%. $\alpha^O = \alpha = 0.07$ case corresponds to the case where both optimists and households are rational, i.e., $\mathbb{E}_t^O = \mathbb{E}_t$.

	l	δ^O	δ^H	ρ	χ	σ	k	α	α^O set
Values	0.4	0	0	0.03	1	0.08	18	0.07	[0.07, 0.13, 0.19]

Table 1: Parameterization for $\alpha^O \geq \alpha$

2.5.1 Risk Amplification from the Optimism

Figure 1 presents the equilibrium quantities of the model for different levels of optimism.[21] In the economy's stochastic steady state, all capital is operated by the optimists who are sufficiently capitalized (i.e., η_t is high). Since the optimists have a superior ability to manage capital, this 'efficient region' of the economy features a high capital price with low endogenous volatility. The q-theory result implies that investment rate is high, and that in turn leads to higher growth rates in both capital and output. A series of negative shocks then drains the net worth of optimists since they operate with leverage, as seen in Figure 1b. Once the wealth share falls sufficiently below a threshold, say η^ψ, optimists start fire-selling capital to the inefficient agents (i.e., rational households) who demand a high risk premium to hold the risky asset since they are not as productive in operating the capital as the optimists. This inefficient "crisis" region features low capital price and investment, high endogenous volatility, and a low output. These features are common to many macro-finance models, e.g., Brunnermeier and Sannikov (2014, 2016b). The role of optimism in our model is to change the equilibrium allocation in two ways: by affecting the threshold at which optimists fire-sell capital, and the amplification that the fire-selling generates. The effect of optimism is particularly salient in the crisis region with little effect around the stochastic steady state. For example, optimism $\alpha^O - \alpha$ does not affect the price of capital q_t in the steady state since optimists operate all capital and the 'risk-adjusted' return that the households obtain from holding capital is less than the prevailing risk-free rate.[22] Thus

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[21]Figure A1 in Appendix A provides other equilibrium quantities not included in Figure 1.

[22]We observe in Figure 1d that the *true* risk premium is negative around the stochastic steady state, which aligns with the fact that households do not hold capital at all in those regions. In contrast, the *perceived* risk

they prefer to invest in risk-free bonds issued by optimists. However, in the inefficient region where less productive households hold a portion of capital, the degree of optimism $\alpha^O - \alpha$ starts to matter. As the optimism increases, the threshold for the efficient region η^ψ falls. That is, a higher optimism leads to a larger part of the state space where optimists operate all capital by taking on more leverage. In particular, the fact that they take on more leverage in the normal region precipitates to a stronger fire-selling effect as evidenced by a steeper slope of capital price as seen in Figure 1a, characterizing the boom-bust cycles documented in the literature (see Krishnamurthy and Li (2020), for example).

(a) Capital price q_t

(b) Leverage multiple x_t

(c) Endogenous volatility σ_t^p

(d) *Perceived* and *true* risk-premium
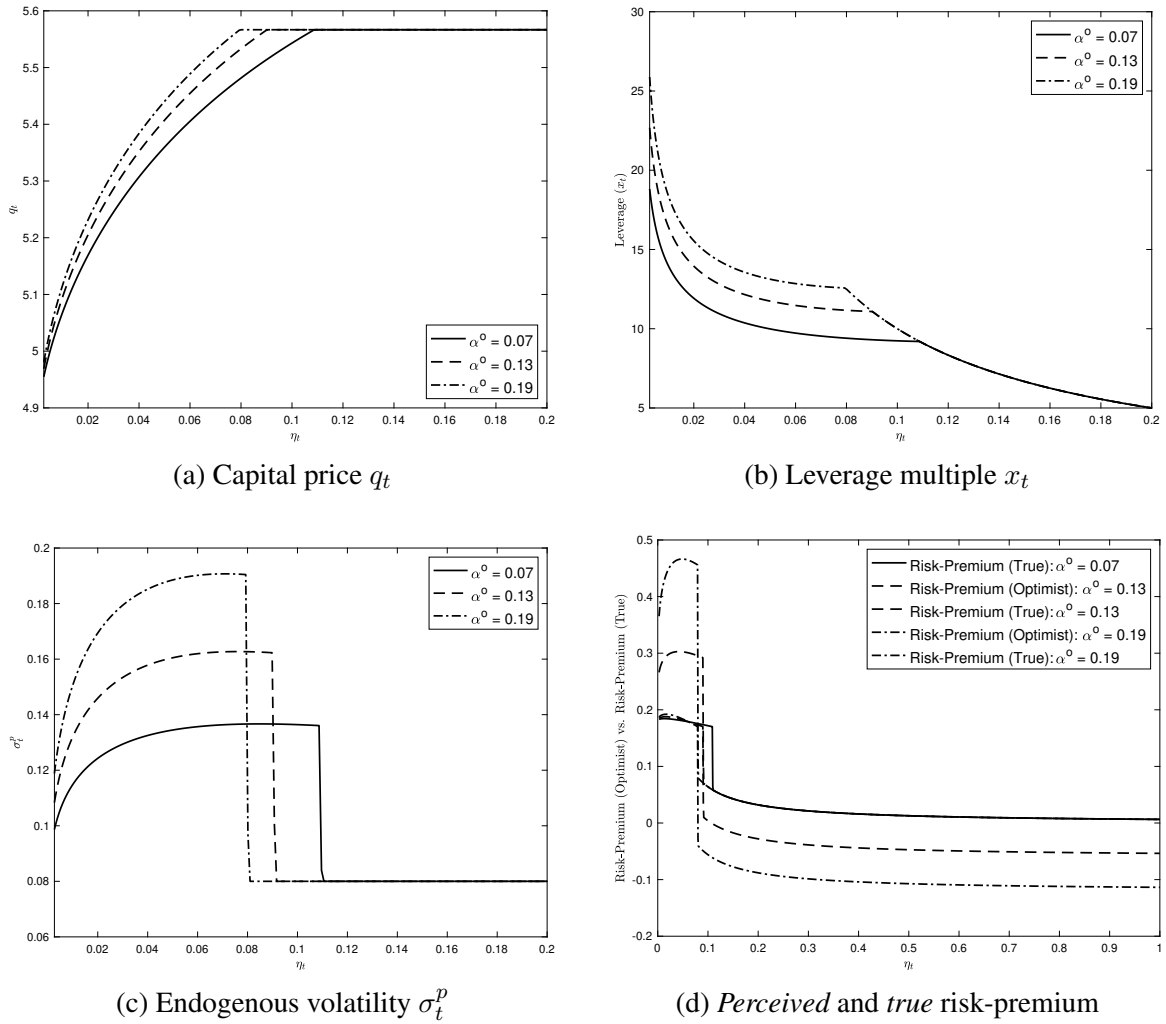
Figure 1: Equilibrium variables as functions of η_t

The role of optimism as risk amplifiers on top of the financial accelerator role can be

premium of optimists is positive in both regions.

seen by inspecting the endogenous volatility of price σ_t^p shown in Figure 1c. As optimism increases, the endogenous volatility σ_t^p is more amplified in the crisis region. Actually, the endogenous risk can be written as

$$\sigma_t^p \left(1 - (x_t - 1) \frac{\frac{dq(\eta_t)}{q(\eta_t)}}{\frac{d\eta_t}{\eta_t}} \right) \equiv \sigma_t^p \left(1 - (x_t - 1) \varepsilon_{q,\eta} \right) = \sigma, \tag{36}$$

where $\varepsilon_{q,\eta}$ is defined as the elasticity of the price-earnings ratio (i.e., the normalized capital price) with respect to the optimists' wealth share η_t. We observe that optimism raises the price volatility through a higher leverage (*leverage effect*). Second, a greater optimism leads to a higher price elasticity (*elasticity effect*), captured by $\epsilon_{q,\eta}$, and magnifies the leverage effect. The elasticity effect arises due to the intense fire-selling of optimists explained earlier. In short, a higher optimism causes a boom phase of excessive leverage, which leads to a strong fire-sales of risky capital to the inefficient agents, both of which contributing to a higher price volatility σ_t^p in the economy's crisis region. One point worth noting is that we capture a belief-driven adverse loop: a higher optimism increases endogenous risk σ_t^p in a crisis, raising the degree of optimism in the expected capital return as seen in (15), which in turn amplifies the endogenous risk even further.

Lastly, when optimists are sufficiently capitalized, a greater optimism leads to a lower interest rate, as seen in Figure A1c of Appendix A.2. In the crisis region where optimists fire-sell capital to households, a higher optimism creates a stronger precautionary saving motive due to a greater endogenous volatility, pushing down the risk free rate.[23] Most of the effects explained thus far stem from the fact that the risk premium perceived by optimists are different from the true risk premium of capital. This expectation error affects the net worth of the optimists strong enough to generate an amplified boom-bust cycle. Optimism makes the productive agents in the economy to take on excessive leverage precisely because they perceive the risk premium to be higher than it actually is.

The expectation error stemming from optimism can drain the net worth of optimists at the stochastic steady state, and raise the occupation time of economy in a crisis. In fact, the static comparative plot in Figure 1 reveal that there is a smaller measure of the state space

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[23]When optimists start deleveraging and fire-sell their capital assets, the interest rate r_t drops in a discontinuous manner. It is anticipated: as optimists suddenly fire-sell the capital and reduce their leverage demand, the risk-free rate should drop to clear the bond market. After the economy enters the inefficient region, i.e., $\eta_t \le \eta^\psi$, the interest rate level becomes continuous again.

where capital is misallocated to unproductive households from fire-sales, when optimism is higher. However, since the expectation errors are high with optimism, it drains the net worth of optimists at a high rate, trapping their wealth share at low levels for a long time. This important channel explains a novel phenomenon called as 'net-worth trap' which we explain next by characterizing the stationary distribution of the optimists' wealth share.

Net worth trap   Figure 2 draws the stationary distribution of η_t in the presence of optimism: we observe that under the benchmark rational expectations model, i.e., $\alpha^O = \alpha$, the time the economy spends the most is around its stochastic steady state.[24] We observe that optimism generates a larger density mass in the crisis region, with sharper peaks inside the crisis region, especially around $\eta \simeq 0$. When negative shocks shift the economy toward the inefficient region, a higher optimism, on average, makes the economy more vulnerable to negative shocks through its effects on leverages, increasing its occupancy time in a crisis. In fact, even moderate levels of optimism generates this effect in which all of the capital is allocated to inefficient agents in the economy for most of the time. When optimism is high beyond a threshold, in addition to this inefficient allocation, the probability of optimists to rebuild their net worth becomes zero. We call this phenomenon a "net worth trap", since the economy gets trapped in this state and remains perennially inefficient.

The origins of the net worth trap lies in the endogenous risk and expectation errors of optimists. There are two opposing forces in crisis that affects the net worth of optimists. First, risk premium (both true and perceived) is higher in a crisis, which loads positively on the expected growth rate of optimists' net worth. This is because optimists are leveraged in risky capital and earn a premium for holding it, helping them to recapitalize faster. The other force is the expectation error from optimistic beliefs about the long run growth rate of productivity. When the optimism α^O is high enough, the second expectation error channel dominates the first recapitalization channel. This effect drains the net worth of experts and (i) increases the likelihood of fire-selling capital in the steady state, and (ii) increases the occupancy time in a crisis. The effect of this expectation error on the drift of wealth share of optimists can be seen in Figures 3a. The drift $\mu^\eta(\eta)\eta$ becomes more negative in the efficient region. Moreover, its endogenous risk, captured by $\sigma^\eta(\eta)\eta$, increases in optimism during

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[24]In generating Figure 2, we introduce the exogenous exit of optimistic experts, as otherwise we obtain the degenerate steady state distribution at $\eta_t = 1$ in the rational expectations benchmark when the discount rates for optimists and households are equal. We use the exit rate$= 7.5\%$. Other than generating a well-behaved stationary distribution, the introduction of exits of optimists has no effect on other equilibrium properties. For this issue, see e.g., Gopalakrishna (2022).
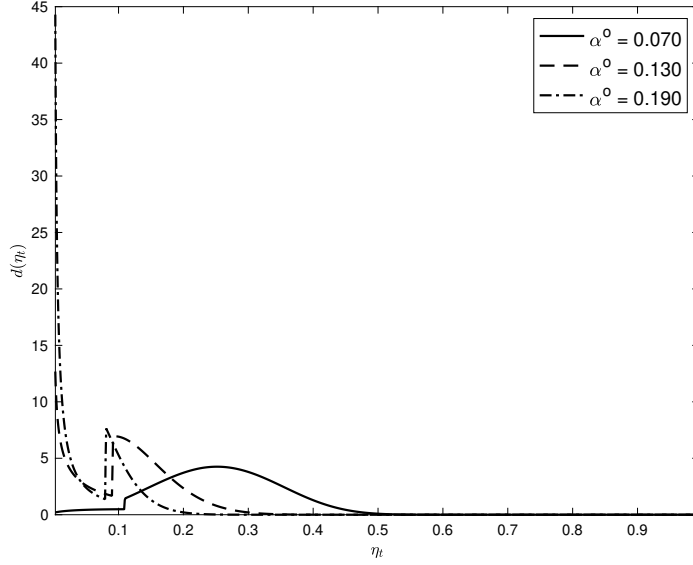
Figure 2: Stationary distribution of η_t and the net worth trap

a crisis, but does not change with optimism in the efficient region, as shown in Figure 3b. The growth rate of optimists' net wealth share (i.e., drift) in the efficient region relative to its endogenous volatility is a crucial variable determining the probability of recapitalization for optimists.

We next show even for a moderate level of optimism, the growth rate of the optimists' wealth share relative to its endogenous volatility becomes low enough (due to high expectation errors) to create a two-peaked stationary distribution with an infinite mass at $\eta = 0$. For high optimism levels, the *relative* growth rate becomes low enough to force the probability of recapitalization of optimists to zero, as documented by the next Proposition 4.

Proposition 4 *There exists a threshold level of optimism $\bar{\alpha}^O$ above which the economy is trapped at $\eta = 0$, and the probability of recapitalization for optimists goes to zero.*

Proof. We analyze the tail of the stationary density to pin down a threshold value of optimism that generates a net worth trap. From (B.35) in Appendix B.2, the asymptotic solution for the stationary density $d(\eta)$ when $\eta \sim 0$ is given by

$$d(\eta) \sim \left(\frac{2\mu^\eta(0)}{(\sigma^\eta)^2(0)} - 1 \right) \eta^{\frac{2\mu^\eta(0)}{(\sigma^\eta)^2(0)} - 2} \tag{37}$$

where the ratio $\tilde{D}_0 \equiv \frac{2\mu^\eta(0)}{(\sigma^\eta)^2(0)}$ determines the existence of a degenerate distribution around $\eta \sim 0$. If $\tilde{D}_0$ goes below 2, then the stationary distribution $d(\eta)$ becomes degenerate (i.e.,
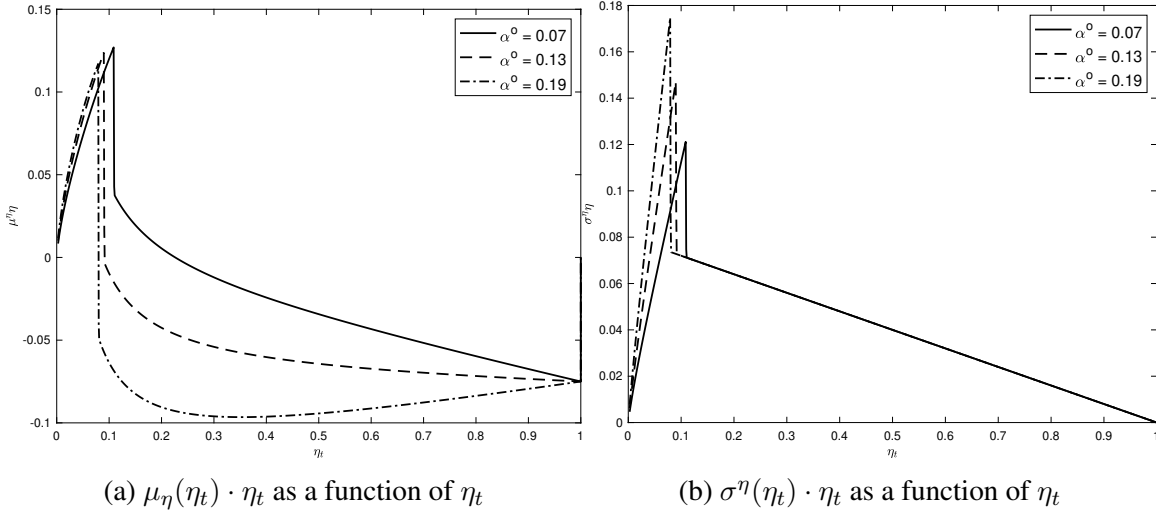
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(a) $\mu_\eta(\eta_t) \cdot \eta_t$ as a function of η_t (b) $\sigma^\eta(\eta_t) \cdot \eta_t$ as a function of η_t

Figure 3: Wealth share dynamics: drift and volatility

Dirac-delta) around $\eta = 0$.[25]

Figure (4a) plots $\tilde{D}_0$ against the optimists' perceived long-run growth rate α^O when the true growth rate is $\alpha = 0.07$. With pessimism or rationality, i.e., $\alpha^O \leq \alpha$, $\tilde{D}_0$ is far above 2, indicating a non-degenerate stationary distribution with finite mass at the point $\eta \sim 0$. In this case, even when the economy enters a crisis, the relative growth rate of optimists' net worth compared with its volatility is high (i.e., expectation errors are small) enough to enable optimists to recapitalize. Under higher optimism, the quantity $\tilde{D}_0$ becomes smaller than the threshold value of 2, indicating that the density features infinite mass at the point $\eta \sim 0$. Once the optimism crosses a threshold value $\bar{\alpha}^O$, $\tilde{D}_0$ falls below 1, where the distribution becomes a Dirac-delta measure. In this case, the relative growth rate of optimsts' wealth share compared with its volatility is negative (i.e., expectation errors are large) such that once the economy enters a crisis, the negative growth rate drains the wealth share until it reaches the point $\eta = 0$. At this point, the optimists can no longer recapitalize, and the economy gets trapped in this state inefficiently forever. ∎

Corollary 1 *The asymptotic skewness of the stationary distribution $d(\eta)$ increases in optimism α^O.*

We can analyze the third moment of the distribution $d(\eta)$ when η approaches 0. Figure 4b draws $\log$-skewness of the distribution around $\eta \sim 0$. We observe that the skewnewss rises with the experts' optimism measure α^O, which is confirmed by Corollary 1.

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[25] $\mu^\eta(0) \equiv \lim_{\eta \to 0} \mu^\eta(\eta)$ and $\sigma^\eta(0) \equiv \lim_{\eta \to 0} \sigma^\eta(\eta)$ in (37) are derived in Appendix B.2.

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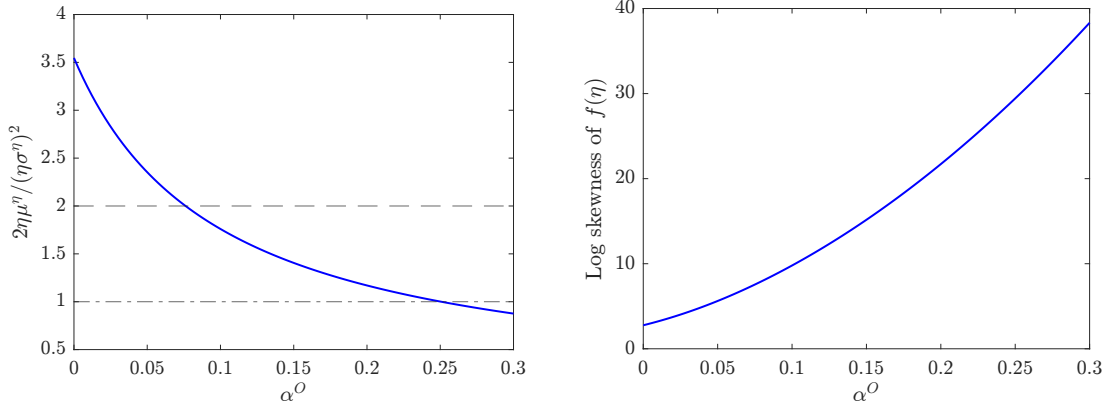
(a) Tail analysis of stationary distribution   (b) Skewness of the distribution around $\eta \sim 0$

Figure 4: The dashed line in Figure 4a indicates the threshold value of 2 below which the economy generates a net worth trap.

Next, we analyze the welfare implications of the optimism on the rational agents in the economy.

2.5.2 The Household's Welfare

We compute the welfare loss of the rational households due to the optimism of experts in the economy as follows:

$$\text{Welfare Loss} = \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log c_t^H dt \right] - \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log c_t^{H,REE} dt \right] \qquad (38)$$

where $c_t^{H,REE}$ is the household's consumption in the rational expectation benchmark. The welfare of the households can be decomposed as

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log c_t^H dt \right] = \underbrace{\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log(1 - \eta_t) dt \right]}_{\text{Wealth effect}_+} + \underbrace{\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log(1 - \iota_t) dt \right]}_{\text{Investment effect}_+}$$

$$(39)$$

$$+ \underbrace{\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log K_t dt \right]}_{\text{Capital effect}_-} + \underbrace{\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log A(\psi) dt \right]}_{\text{Misallocation effect}_-} + \underbrace{\text{t.i.e.}}_{\text{Terms independent of equilibria}}$$

where $A(\psi) = \psi_t + l(1 - \psi_t)$ is the aggregate (i.e., weighted average of capital share based on productivities) capital share in the economy, and $(1 - \eta_t)$ captures the wealth share of the

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households. The wealth effect is an obvious direct effect that arises from a higher wealth share of households in the presence of optimism. Since the expectation errors particularly affect the optimists by draining their net worth, the share of wealth held by households is higher. A higher occupancy time at the crisis due to optimism means that the investment rate is lower, leaving the households to consume more. However, there are other general equilibrium effects that arise out of a higher occupancy time in a crisis due to optimism. First, the aggregate capital stock will be lower in the future as well due to a lower growth rate of capital. Similarly, the share of capital held by households, who are less productive than optimists, gets bigger, captured by the misallocation effect. The last two components have a negative effect on the welfare. We perform numerical simulations and show that the negative general equilibrium effects dominate the direct wealth and investment effect, leading to an overall decline in welfare. In fact, all these forces become stronger as optimism increases, and thus the total welfare of rational agents decrease in optimism as displayed in Figure 5.[26]
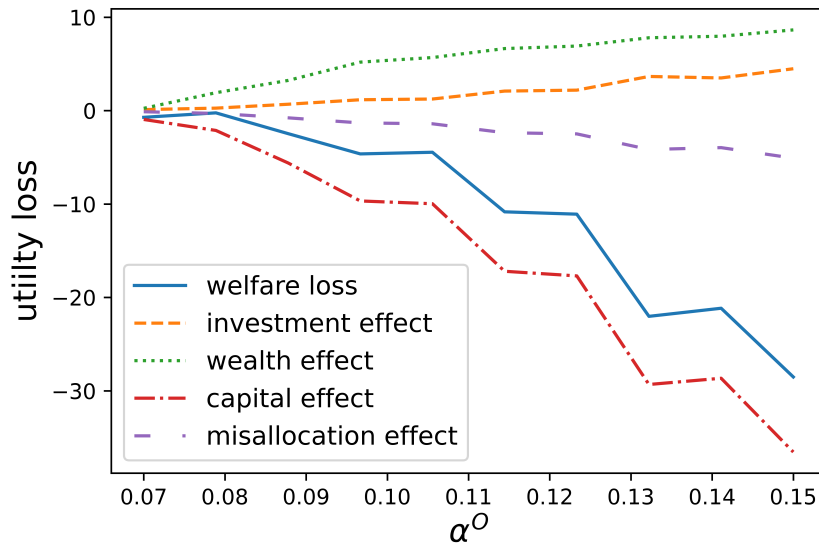

Figure 5: Decomposition of the rational household's welfare loss

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3 Quantitative Analysis

In this section, we perform a quantitative analysis to test a couple of basic model predictions. One prediction of the model is that optimism exacerbates financial instability, and leads to a time varying risk premium on the capital stock. In particular, the risk premium increases in crisis as a result of the feedback loop between optimism and the wealth share of productive agents. A static comparison shows that larger optimism leads to persistently high risk premium through higher occupancy time in crisis. We turn to testing this prediction first before moving on to the cross-sectional implications of optimism.

3.1 Time-series Return Predictability

To study the time-series return predictability patterns of asset returns, we regress the excess return on the S&P500 on its lagged excess return controlling for recession periods.

$$r_{t+h}^e = \alpha(h) + \beta_1(h)r_t^e + \beta_2(h) \times r_t^e \times 1_{Rec} + \epsilon_{t+h} \tag{40}$$

The recession indicator 1_{Rec} takes a value 1 during the NBER recessionary months, and 0 otherwise. The coefficient $\beta_2(h)$ measures the excess conditional momentum. Figure (6) displays the empirical auto-correlation coefficients. Empirically, the aggregate stock return exhibits a strong momentum in the shorter horizons, and reversal over horizon 5 months. This effect is stronger in the crisis period which can be seen in the left panel, corroborating with Cujean and Hasler (2017) who find that the return predictability is concentrated during recessionary periods. To compute the model implied correlation coefficients, we first construct the excess return over a period of length Δ based on the following definition.[27]

$$R_t^e = \int_{t-\Delta}^t \left(\frac{d(q_u K_u) + (\hat{A}_u - \iota_u)K_u du}{q_u K_u} - r_{f,u}du \right) \tag{41}$$

We then simulate the model 1000 times for 5000 years and compute the average slope cofficients from the following regression

$$R_{t+h}^e = \alpha(h) + \beta_{1,model}(h)R_t^e + \beta_{2,model}(h) \times R_t^e \times 1_{crisis} + \epsilon_{t+h} \tag{42}$$

If the sign of coefficients is positive (negative), the return exhibits momentum (rever-

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[27]The scaled aggregate productivity $\hat{A}_t := \frac{A_t}{\gamma_O}$ is given by $\hat{A}_t = A(\psi_t) = \psi_t + l(1 - \psi_t)$.

sal). The bottom two graphs in the Figure (6) display the model-implied correlation coefficients. The model captures the 'excess' short-term momentum and long-term momentum crash quite well.[28] After a series of negative shocks, the economy is dragged into a crisis period with depressed wealth share of optimists and high expected returns. An increase in risk premium during the recapitalization phase leads to the conditional momentum in asset returns. The optimists recapitalize until the economy transitions back to the efficient region where the expected returns are low, leading to momentum crash. While the empirical conditional momentum crashes after a few lags, the model implied conditional momentum persists for several periods. We next turn to analyse the role of macro level optimism in the excess conditional momentum pattern.
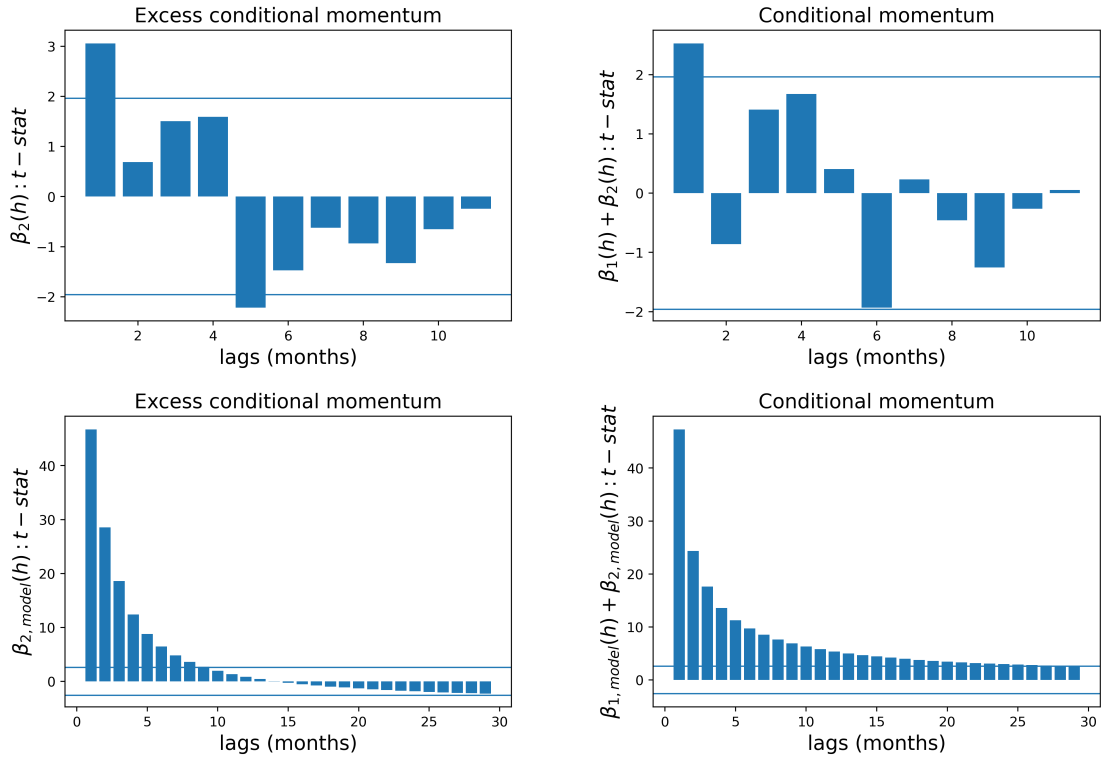

Figure 6: Time series return predictability. Top two panels present the empirical autocorrelation coefficients from regressing the excess return on the S&P500 on its lagged excess return, as shown in the equation (40). The data is at monthly frequency from 1945 till 2022. The bottom two panels present the model implied autocorrelation coefficients from the regression (42). The model is simulated 1000 times for 5000 years at a monthly frequency. Each correlation coefficient represents the average value across simulations.

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[28]We define the 'excess' momentum/reversal to be the momentum/reversal in crisis period relative to the normal period.

3.1.1 Impact of Optimism

To study the impact of macro-optimism on conditional return predictability patterns empirically, we run the following regression:

$$r_{t+h}^e = \alpha(h) + \beta_1(h)r_t^e + \beta_2(h) \times r_t^e \times 1_{Rec} + \beta_3(h) \times r_t^e \times 1_{Rec} \times 1_o + \epsilon_{t+h} \qquad (43)$$

where 1_d is a dummy variable that takes a value of 1 if the macro-optimism based on The Survey of Professional Forecasters O_t that is defined in (46) in Section 3.2 is above its median. The coefficient $\beta_3(h)$ measures the excess conditional momentum when optimism is high. We run a similar regression in our model with optimism proxied by the 'net leverage' defined by

$$x_t^{net} = x_t - x_t^{REE}, \qquad (44)$$

which captures the component of the optimists' total leverage attributed to optimism. x_t^{REE} is the leverage of experts under the rational expectations equilibrium. While the optimism of experts about the expected technological growth, i.e., $\alpha^O - \alpha$ is constant in our model, it triggers a feedback loop between the experts' optimism and their wealth share, impacting their leverage choices. It implies that the net leverage, a component of leverages generated by the optimism, is time-varying. In the model regression, the dummy variable 1_o takes a value 1 if the net conditional leverage of optimists is larger than its median value in each simulated path.[29] Figure (7) presents the results from the regression. The model successfully captures the excess conditional momentum during high optimism periods, which is evident in the data. Higher optimism increases the vulnerability of the economy in stochastic steady state. Thus, an adverse shock might lead to a crisis with high expected returns on capital. With stronger optimism, the expectation error increases the likelihood of a net worth trap leading to an elevated risk premium. That is, the feedback loop between the net worth and capital misallocation keeps the conditional risk premium higher for a long time, leading to even higher excess conditional momentum. After a few months, the momentum crashes since positive shocks erode the expectation errors eventually helping the experts to recapitalize. The economy thus transitions into the efficient region, where the actual risk premium falls . Next, we turn to cross-sectional implications of macro-level optimism.

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[29]The reason we use conditional leverage is because in our model, optimism affects equilibrium quantities only during crisis.
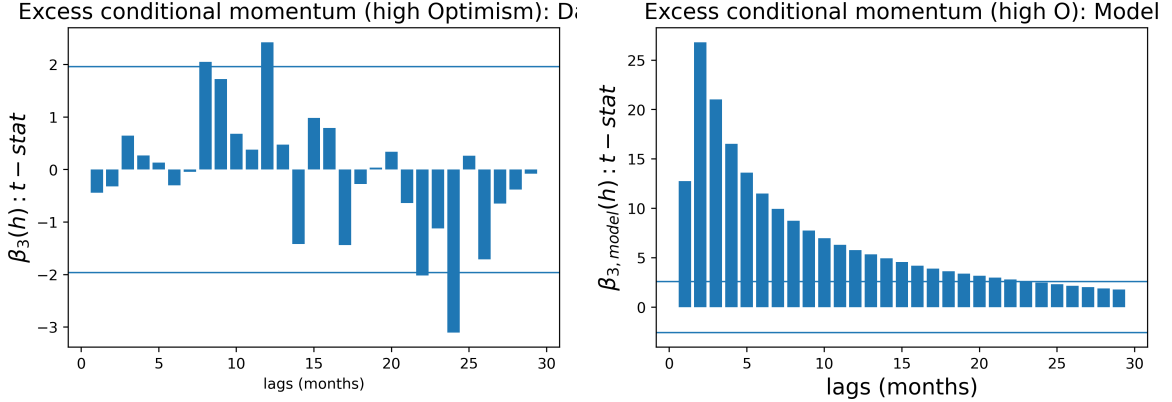
Figure 7: Time series return predictability. The left panel presents empirical autocorrelation coefficients from regressing the excess return on S&P500 on its lagged excess return, as shown in equation (43). The data is at monthly frequency from 1945 till 2022. Macro-optimism is computed from the definition (46). The macro-optimism data is available at a quarterly frequency and hence interpolated to get monthly values. The left panel presents the conditional t-stats when the optimism is high ($\beta_3(h)$). The right panels presents the model-implied conditional t-stats when the optimism is high ($\beta_{3,model}(h)$).

3.2 Factor Model

Our model predicts that asset returns are affected by a two-way feedback loop between optimism and the wealth share of experts. The prediction naturally leads to a factor pricing model with a role for both optimism, and the intermediary risk capacity as a proxy for the wealth share. In that spirit, we propose a three factor model with macro-optimism along with the aggregate wealth and the intermediary wealth. The intermediary wealth is proxied by the intermediary capital ratio from He et al. (2017). That is, the capital ratio, denoted by η_t is the fraction of total assets held in equity by the bank holding companies in the US:

$$\eta_t \equiv \frac{\text{Equity}_t}{\text{Assets}_t} \tag{45}$$

The aggregate wealth W_t is proxied by the traditional market factor. The third novel factor, macro-optimism, is based on The Survey of Professional Forecasters and empirically measured as

$$O_t = \frac{f_{75}}{|f_{50}|} \tag{46}$$

where f_i denotes the ith percentile analyst forecast of quarter-on-quarter GDP growth rate for the $(T+2)^{th}$ quarter ahead at date T. We run a two-stage Fama-McBeth regression with

the three factors $f_t := [M_t, \eta_t, o_t]'$, where M_t is the market excess return, and o_t is a percent change in $1 + O_t$. First, we regress the time series excess return of each asset i on the three factors to estimate corresponding betas $\hat{\beta}_{i,t}$ using the following regression equation:

$$R_{i,t}^e = a_i + \beta_{i,f}' f_t + v_{i,t} \tag{47}$$

In the second stage regression, the estimated betas, i.e., $\{\beta_{i,f}\}_i$, are used to obtain the risk prices for each of the factors by the following regression

$$E[R_{i,t}^e] = \alpha_i + \hat{\beta}_{i,f}' \lambda_f + \varepsilon_i \tag{48}$$

where $E[R_{i,t}^e]$ is the excess return on the asset i, and λ_f is the risk price of the corresponding factor $f \in f_t$. We adjust the standard errors of the estimated λ_f in order to account for the fact that regressors $\hat{\beta}_{i,f}$ are estimated quantities.

Test assets: To test the ability of macro-optimism in pricing the cross-section of asset returns above and beyond the intermediary factor and the market factor, we use the following test assets from the period 1970Q1 till 2022Q4: 25 size and book-to-market portfolios, 25 size and momentum sorted portfolios, 10 long-term reversal portfolios, 25 profitability and investment portfolios, and 10 maturity sorted US treasury bond portfolios from CRSP Fama bond dataset with maturities in six month intervals up to five years. Using a large number of test assets passes the criticism by Lewellen et al. (2010). In addition to the equity and bond portfolios used above, we extend the analysis to include other asset classes with the following test assets from 1970Q1 till 2012Q4 (due to data availability) - 18 option portfolios, 20 CDS portfolios, and 12 FX portfolios used in He et al. (2017).

Table (2) presents the results from the first-stage regression using the equity and bond portfolios, as well as HKM test assets including momentum and long-term reversal portfolios as test assets. The table displays the following statistics for the two different asset classes: average excess returns, standard deviations of the excess returns in each asset class, mean and standard deviation of the factor exposures of the excess returns to the factors f_t. We consider two models for each asset classes: a two-factor model with the market and the intermediary factors, and a three-factor model with the market, intermediary, and macro-optimism factors. The regression controls for the price-dividend ratio, cyclically adjusted price-earnings ratio (CAPE) obtained from Robert Shiller's website, and VIX on S&P500 index from CBOE. The first two variables are commonly used to predict asset returns, and

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we control for VIX to condition on aggregate asset return volatility. Consistent with He et al. (2017), there is a large variation in the intermediary factor exposure in both asset classes.[30] In addition, exposures to the macro-optimism factor also show a considerable variability in both asset classes. That is, the standard deviation of β_O is around twice its mean, indicating that the excess returns of test assets have sufficient variation in its exposure to the macro-optimism factor in (46).

	Equities and Bonds		HKM + Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Mean excess return	1.88	1.88	1.18	1.18
Std. excess return	0.84	0.84	1.32	1.32
Mean β_M	0.9	0.9	0.55	0.55
Std β_M	0.37	0.37	0.46	0.46
Mean β_η	0.08	0.08	0.07	0.0
Std β_η	0.11	0.11	0.13	0.13
Mean β_O	-	0.004	-	-0.04
Std β_O	-	0.04	-	0.08
Assets	95	95	95	129
Quarters	211	211	211	171
Controls	Yes	Yes	Yes	Yes

Table 2: Expected returns and risk exposures. Equity assets include 25 size and book-to-market portfolios, 25 size and momeutum portfolios, 10 long-term reversal portfolios, and 25 profitability and investment portfolios. Bond portfolios include 10 maturity sorted portfolios from CRSP Fama bond portfolio dataset. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from He et al. (2017). HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. Mean and standard deviation of the excess return is the difference in mean return and risk free rate of the corresponding test assets. The frequency is quarterly and time period is from 1970Q1 till 2022Q4. The mean and std. of betas ($\beta_M, \beta_\eta, \beta_O$) measure the average and standard deviation of the exposure of excess return to the market factor, the intermediary capital ratio, and the macro-optimism factor respectively.

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[30]Later, we control for the cay measure of Lettau and Ludvigson (2001), and the capital share of Lettau et al. (2019). Since the capital share data are available until 2013Q4, we drop this from our main regression. However, as shown in the robustness section in appendix (A.2), the results are unchanged by the inclusion of these control variables.

3.3 Cross-sectional Asset Pricing Test

Table (3) presents the risk price estimates for both the two-factor and the three-factor models along with Shanken-corrected t-stats. The risk price of the macro-optimism factor in the three-factor model is positive and statistically significant in both equity and bond portfolios. The inclusion of our macro-optimism factor increases the adjusted R-squared significantly by 18% in the case of equity and portfolios. The intermediary factor, although has a positive price of risk estimate, is not statistically significant.[31]

	Equities and Bonds		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Market	0.01	0.01	0.02	0.02
t-stat Shanken	(1.17)	(1.20)	(1.53)	(1.42)
Intermediary	0.02	0.02	0.03	0.04
t-stat Shanken	(1.08)	(1.04)	(1.91)	(1.96)
Macro-optimism	-	0.07	-	0.04
t-stat Shanken	-	(2.93)	-	(2.84)
MAPE %	2.22	1.98	2.83	2.79
Adj. R2	0.22	0.40	0.45	0.52
Assets	95	95	129	129
Quarters	211	211	171	171

Table 3: Risk price estimates for equities and US government bond portfolios. Equity and bond test assets include 25 size and book-to-market portfolios, 25 size and momentum portfolios, 10 long-term reversal portfolios, and 25 profitability and investment sorted portfolios, 10 maturity sorted US government bond portfolios taken from the CRSP Fama bond portfolio dataset. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from HKM2017. In addition, it also includes 25 size and momentum portfolios and 10 long-term reversal portfolios. The data is at quarterly frequency from 1970Q1 till 2022Q4. The factors are market, intermediary capital ratio, and macro-optimism. The macro-optimism factor o_t is computed as growth rate in O_t, the 75th percentile of GDP forecast scaled by median forecast by analysts. The forecast data is taken from The Survey of Professional Forecasters. The standard errors of risk price estimates are adjusted for generated regressor problem using Shanken correction. MAPE denotes the mean absolute pricing error in annualized terms.

Figure (8) displays the actual average excess returns against the predicted average ex-

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[31]He et al. (2017) documents that the intermediary risk factor is not significant with 25 size and book-to-market portfolios and 10 momentum sorted portfolios.

cess returns for the two-factor model. The predicted excess returns is from the two-factor model with the market and the intermediary factor. Figure (9) displays the same with predicted excess returns from the three-factor model with macro-optimism. The predicted returns from the three-factor model aligns better along the 45-degree line compared to the predicted returns from the two-factor model. This observation is confirmed with Table (3) that shows that the mean absolute pricing error (MAPE) from the three-factor model is 24 basis points lower than the error from the two-factor model.
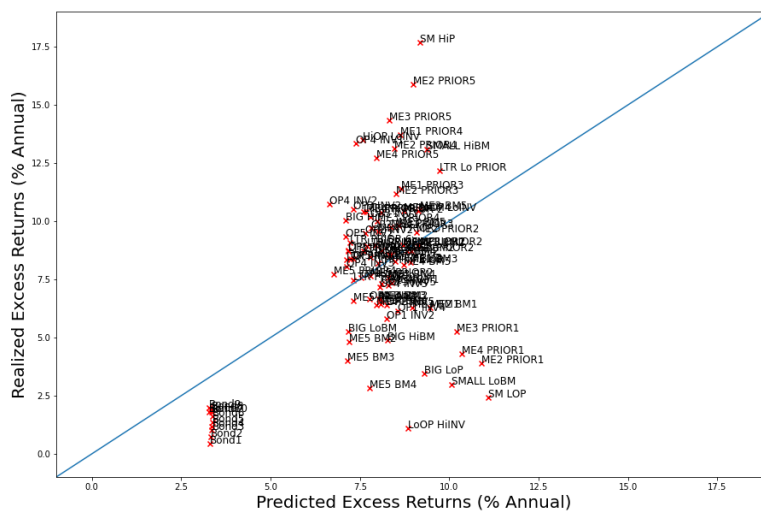

Figure 8: Pricing errors on equity and bond portfolios with two factors: Realized excess returns versus predicted excess returns using the two-factor model with market and intermediary factors. The data is at quarterly frequency and from 1970Q1 till 2022Q4.

Table (3) also presents the risk price estimates from the two-factor and the three-factor models with HKM test assets. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from He et al. (2017). In addition, it also includes 25 size and momentum portfolios and 10 long-term reversal portfolios. The risk price of the macro-optimism factor is positive and statistically significant in both cases. The adjusted R-squared increases by 7% and the mean absolute pricing error decreases by 4 basis points. Figure (10) also displays the average realized excess returns vs. the average predicted excess returns for HKM+Momemtum assets. The predicted returns in the three-factor model more line up towards the 45-degree line than in the two-factor model, which is
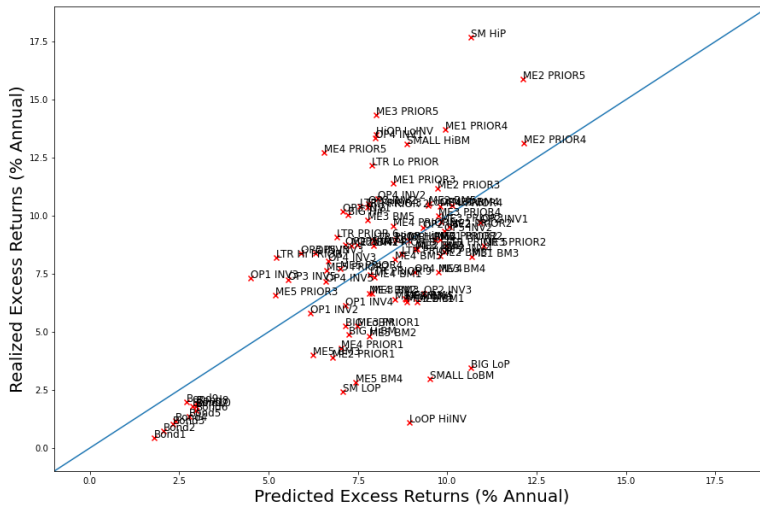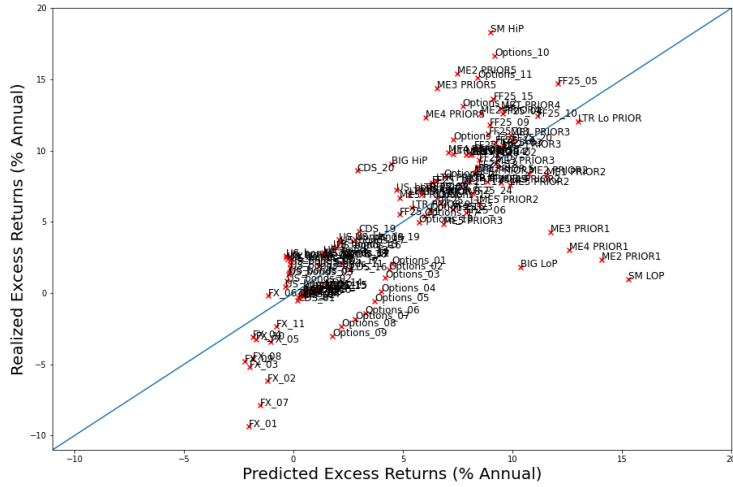
Figure 9: Pricing errors on equity and bond portfolios with three factors: Realized excess returns versus predicted excess returns using the three-factor model with market, intermediary, and macro-optimism factors. The data is at quarterly frequency and from 1970Q1 till 2022Q4.

consistent with the reduction in pricing error shown in the table (3). Overall, the evidence for the macro-optimism factor in pricing the cross-section of asset returns above and beyond the intermediary and the market factor is strong for equity, bond, and other asset classes.

		Macro-optimism			
		(1)	(2)	(3)	(3)-(1)
Intermediary	(1)	3.25	3.61	6.12	2.86
	(2)	7.50	5.03	7.76	0.26
	(3)	8.77	9.65	9.37	0.59
	(3)-(1)	5.52	6.04	3.25	-

Table 4: Average excess returns. The table reports the annualized mean excess return on equity and bond portfolios double-sorted on their exposures to the intermediary factor and the macro-optimism factor using the three-factor model. The data is at quarterly frequency from 1970Q1 till 2022Q4. The intermediary factor is from HKM2017, and the macro-optimism factor is computed from the growth rate of 75th percentile GDP projection, scaled by the median projection.

(a) Pricing error in two-factor model.
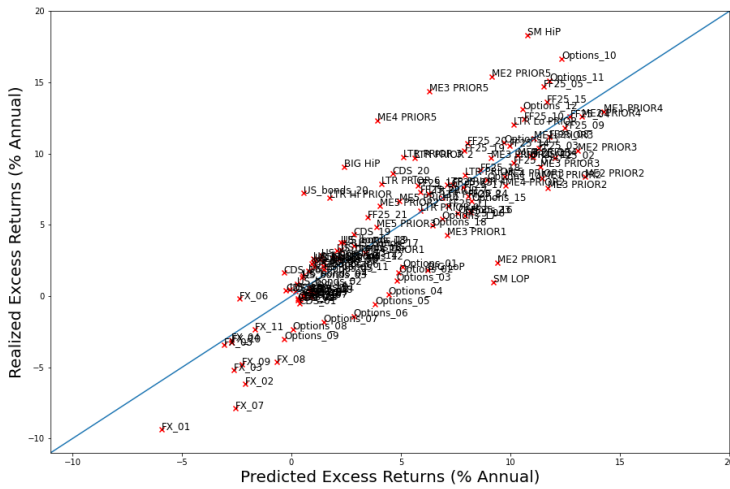
(b) Pricing error in three-factor model.

Figure 10: Pricing errors on HKM+Momentum portfolios. Realized excess returns versus predicted excess returns using the two-factor model with the market and the intermediary factor in panel (a), and the three-factor model with the market, intermediary, and macro-optimism factors in panel (b). The data is quarterly and from 1970Q1 till 2012Q4.

3.4 Two-way Sorted Portfolios

To further verify the positive risk price of macro-optimism, we independently double sort the test assets based on their exposures to the intermediary factor and the macro-optimism factor, controlling for the market factor, price-dividend ratio, cyclically adjusted earnings ratio (CAPE), and the cay variable of Lettau and Ludvigson (2001). A rolling-window of 60 periods is used to estimate the betas and for sorting the assets. Table (4) presents the average excess returns for three-by-three double-sorted portfolios. The high-minus-low return of the portfolios sorted with respect to the macro-optimism beta, i.e., $\beta_{i,o}$, is consistently positive in all three tertiles as seen in the last column of table (4). The economic magnitude of the high-minus-low returns is also large with upto 2.86% in annualized terms. Thus, the macro-optimism factor has pricing power beyond the market and the intermediary factor.

4   Conclusion

In this paper, we build a simple continuous-time economy with optimistic experts and rational households. Experts who are optimistic about the economy's long-term growth raise the probability that the economy enters a crisis, amplify the endogenous risk during crises, and generate a net worth trap: a phenomenon where their wealth share is trapped around low levels for a long duration. Beyond a threshold level of optimism, their probability of rebuilding wealth becomes zero and the economy suffers from perennial inefficiency. This can be attributed to the fact that optimistic experts take excessive leverage, based on an incorrect belief about the growth rate of economy. This expectation error prevents their net worth from getting recapitalized, increases the financial instability, and generates a protracted crisis. Our model quantitatively explains the empirical conditional momentum concentrated during crises, which becomes stronger with optimism. Lastly, we show that a factor pricing model with both the intermediary and the macro-optimism factor based on the Survey of Professional Forecasters (SPF) significantly improves the pricing power in the cross-section of asset returns.

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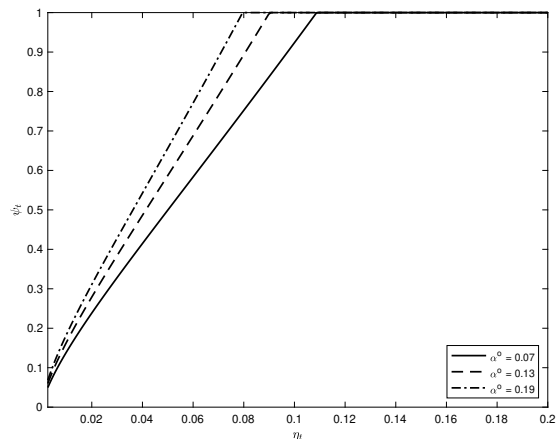
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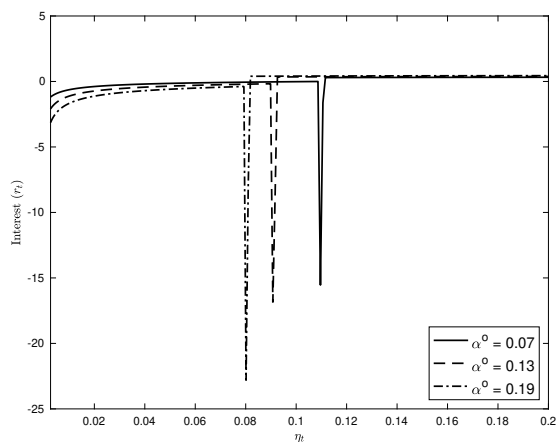
Appendix A Appendix

A.1 Additional Figures
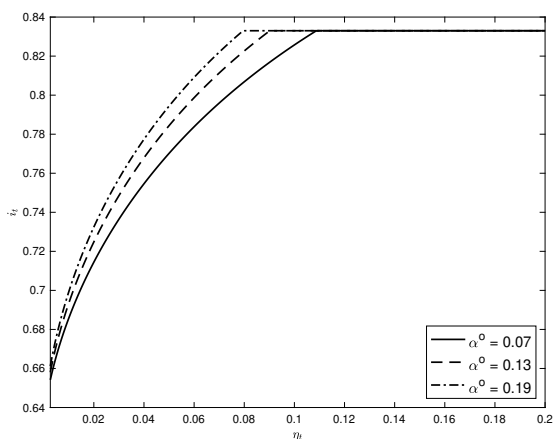
(a) Investment rate ι_t as a function of η_t

(b) Capital share ψ_t

(c) Equilibrium interest rate r_t^*

Figure A1: Other equilibrium variables as functions of η_t

A.2 Robustness check

	HKM		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Mean excess return	0.85	0.85	1.2	1.2
Std excess return	1.31	1.31	1.32	1.32
Mean β_M	0.46	0.46	0.62	0.62
Std β_M	0.46	0.46	0.48	0.48
Mean β_η	0.03	0.03	0.04	0.04
Std β_η	0.09	0.09	0.1	0.09
Mean β_O	-	-0.02	-	-0.03
Std β_O	-	0.04	-	0.06
Assets	94	94	129	129
Quarters	195	195	195	195
Controls	Yes	Yes	Yes	Yes

Table A1: Expected returns and risk exposures- Robustness check. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from He et al. (2017). HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. Mean and std. of excess return is the difference in mean return and risk free rate of the corresponding test assets. The frequency is quarterly and time period is from 1970Q1 till 2012Q4. The mean and std. of betas ($\beta_W, \beta_\eta, \beta_O$) measure the average and standard deviation of exposure of the excess return to market factor, intermediary capital ratio, and macro-optimism measure respectively. Controls include price-dividend ratio, cyclically adjusted earnings ratio (CAPE), cay, and capital share risk.

	HKM		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Market	0.02	0.01	0.02	0.02
	(1.6)	(1.32)	(1.83)	(1.36)
Intermediary	0.09	0.10	0.05	0.07
	(4.48)	(3.64)	(3.01)	(2.47)
Macro-optimism	-	0.06	-	0.09
	-	(1.68)	-	(2.89)
MAPE %	1.7	1.49	2.36	1.95
Adj. R2	0.82	0.86	0.60	0.74
Assets	94	94	129	129
Quarters	195	195	195	195

Table A2: Risk price estimates for HKM and HKM+Momentum portfolios- Robustness check. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from He et al. (2017). HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. The data is at quarterly frequency from 1970Q1 till 2012Q4. The factors are market, intermediary capital ratio, and macro-optimism. The macro-optimism factor is computed as growth rate in the inter-quartile dispersion of GDP forecast scaled by median forecast by analysts. The forecast data is taken from The Survey of Professional Forecasters. The standard errors of risk price estimates are adjusted for generated regressor problem using Shanken correction. MAPE denotes the mean absolute pricing error in annualized terms.

Appendix B   Proofs and Derivations

Proof of Proposition 1. We separately solve problems of optimists and households.

Optimists   Since optimists believe that dZ_t^O, not dZ_t, is the true Brownian motion driving the business cycle, they believe the wealth share η_t follows:

$$\frac{d\eta_t}{\eta_t} = \left(\mu_t^\eta(\eta_t) + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^\eta(\eta_t) \right) dt + \sigma_t^\eta(\eta_t)dZ_t^O, \tag{B.1}$$

which is consistent with the true η_t process in (27). Based on Merton (1971), we conjecture her value function $V(\cdot)$ will depend on her own wealth w_t^O and the aggregate wealth share of optimists η_t with the following form:

$$V\left(w_t^O, \eta_t\right) = \frac{\log w_t^O}{\rho} + f\left(\eta_t\right). \tag{B.2}$$

Based on (16) and (B.1), the Hamiltonian-Jacobi-Bellman (HJB) equation for an individual optimist's problem can be written as[1,2]

$$\rho V(\cdot) = \max_{x_t^O, c_t^O, i_t^O} \log c_t^O + \left[w_t^O \left(x_t^O \left(\frac{\mathbb{E}_t^O\left(dr^{Ok}\right)}{dt} \right) + (1 - x_t^O)r_t \right) - c_t^O \right] \frac{dV_t}{dw_t^O}$$

$$+ \frac{\left(x_t^O w_t^O \sigma_t^p\right)^2}{2} \frac{d^2V_t}{d\left(w_t^O\right)^2} + \left(\eta_t \left(\mu_t^\eta(\eta_t) + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^\eta(\eta_t) \right) \right) \frac{dV_t}{d\eta_t} + \frac{\left(\eta_t\sigma_t^\eta(\eta_t)\right)^2}{2} \frac{d^2V_t}{d\eta_t^2}.$$

The first order condition with respect to c_t^O are given by:

$$\frac{1}{c_t^{O*}} = \frac{dV_t}{dw_t^O}\left(w_t^O, \eta_t\right) = \frac{1}{\rho w_t^O}, \tag{B.3}$$

which gives $c_t^{O*} = \rho w_t^O$ at optimum. As every aggregate variable including q_t, i_t^O, μ_t^p, σ_t^p, x_t^O, and r_t will be expressed as functions of the aggregate wealth share η_t, Merton (1971)

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[1]We use the following relation from equation (15):

$$\mathbb{E}_t^O\left(dr^{Ok}\right) = \left(\frac{\gamma_t^O - \iota_t^O\gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p \right) dt = \left(\frac{1 - \iota_t^O}{q_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p \right) dt.$$

[2]Due to the assumed value function in (B.2), we ignore the cross-derivative term of $V(\cdot)$ with respect to w_t^O and η_t.

justifies our choice of value function in (B.2). The first-order condition with respect to x_t^O gives the optimal portfolio choice given in Proposition 1.

To derive i_t^{O*}, we observe that the investment appears only in a intra-temporal manner. Thus the corresponding optimization problem is static and satisfies the first-order condition: $\Lambda^{O\prime}(i_t) = q_t^{-1}$ (i.e., marginal Tobin's q). With $\Lambda^O(i_t)$ from (29), we finally obtain:

$$i_t^{O*}(q_t) = \frac{q_t^2 - 1}{2k}. \tag{B.4}$$

Rational households Since households have the rational expectations, i.e., they believe that dZ_t, not dZ_t^O, is the true Brownian motion driving the business cycle, they believe the wealth share η_t follows:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta(\eta_t)dt + \sigma_t^\eta(\eta_t)dZ_t, \tag{B.5}$$

which is consistent with the true η_t process in (27). Based on Merton (1971), we conjecture her value function $\underline{V}(\cdot)$ will depend on her own wealth $w_t^P H$ and the aggregate wealth share of optimists η_t as the state variable:

$$V^H\left(w_t^H, \eta_t\right) = \frac{\log w_t^H}{\rho} + f^H\left(\eta_t\right). \tag{B.6}$$

Based on (18) and (B.5), the Hamiltonian-Jacobi-Bellman (HJB) equation for an individual optimist's problem can be written as[3]

$$\rho V^H(\cdot) = \max_{x_t^H \geq 0, c_t^H, i_t^H} \log c_t^H + \left[w_t^H\left(x_t^H\left(\frac{\mathbb{E}_t^H\left(dr^{Hk}\right)}{dt}\right) + (1 - x_t^H)r_t\right) - c_t^H\right]\frac{dV_t^H}{dw_t^H}$$
$$+ \frac{\left(x_t^H w_t^H \sigma_t^p\right)^2}{2}\frac{d^2V_t^H}{d\left(w_t^H\right)^2} + \eta_t\left(\mu_t^\eta(\eta_t)\right)\frac{dV_t^H}{d\eta_t} + \frac{\left(\eta_t\sigma_t^\eta(\eta_t)\right)^2}{2}\frac{d^2V_t^H}{d\eta_t^2}.$$

The first order condition with respect to c_t^G are given by:

$$\frac{1}{c_t^{H*}} = \frac{dV_t^H}{dw_t^H}\left(w_t^H, \eta_t\right) = \frac{1}{\rho w_t^H}, \tag{B.7}$$

---

[3]We use the following relation:

$$\mathbb{E}_t^H\left(dr^{Hk}\right) = \mathbb{E}_t\left(dr^{Hk}\right) = \left(\frac{\gamma_t^H - \iota_t^H\gamma_t^H}{p_t} + \Lambda^H(\iota_t^H) - \delta^H + \mu_t^p\right)dt.$$

which gives $c_t^{H*} = \rho w_t^H$ at optimum. As aggregate variables including q_t, i_t^H, μ_t^p, σ_t^p, x_t^H, and r_t are expressed as functions of the aggregate wealth share η_t, Merton (1971) justifies our choice of value function in (B.6). The first-order condition with respect to x_t^H gives the optimal portfolio choice given in Proposition 1.

To derive ι_t^H, we observe that the investment appears only in a intra-temporal manner. Thus the corresponding optimization problem is static and satisfies the first-order condition: $\Lambda^{H\prime}(i_t) = l \cdot q_t^{-1}$. With equation (30) that $\Lambda^H(\cdot) = l \cdot \Lambda^H(\cdot)$, we finally obtain:

$$i_t^{H*}(q_t) = \frac{q_t^2 - 1}{2k}, \tag{B.8}$$

which is the same as $i_t^{O*}(q_t)$ in (B.4).

∎

Proof of Proposition 2. The aggregate wealth of optimists, W_t^O evolves with the process

$$dW_t^O = r_t W_t^O dt + \psi_t p_t K_t \left(dr_t^{Ok} - r_t dt\right) - c_t^O dt, \tag{B.9}$$

where $c_t^O = \rho W_t^O$ holds at optimum. By applying Ito's quotient rule on (26),[4] we have

$$\frac{d\eta_t}{\eta_t} = \frac{dW_t^O}{W_t^O} - \frac{d\left(p_t K_t\right)}{p_t K_t} + \left(\frac{d\left(p_t K_t\right)}{p_t K_t}\right)^2 - \frac{dW_t^O}{W_t^O}\frac{d\left(p_t K_t\right)}{p_t K_t}. \tag{B.10}$$

In addition, from the process (21) of the aggregate capital K_t, we obtain

$$\begin{aligned}
\frac{1}{K_t}\frac{dK_t}{dt} &= \left(\Lambda^O\left(i_t^O\right) - \delta^O\right)\psi_t + \left(\Lambda^H\left(i_t^H\right) - \delta^H\right)\left(1 - \psi_t\right) \\
&= \left(\Lambda^O\left(i_t^O\right) - \delta^O\right) - \left(\delta^H - \delta^O\right)\left(1 - \psi_t\right) - \left(1 - \psi_t\right)\left(1 - l\right)\Lambda^O\left(i_t^O\right),
\end{aligned} \tag{B.11}$$

where we used the property that $i_t^O = i_t^H$ in equilibrium as seen in (B.4) and (B.8). Applying Ito's product rule to the price process (8) and the capital process (B.11), and comparing with (15), we obtain

$$\frac{d\left(p_t K_t\right)}{p_t K_t} = dr_t^{Ok} - \frac{1 - \iota_t^O}{q_t}dt - \left(\delta^H - \delta^O\right)\left(1 - \psi_t\right)dt - \left(1 - \psi_t\right)\left(1 - l\right)\Lambda^O\left(i_t^O\right)dt. \tag{B.12}$$

---

[4]Ito's quotient rule states that:

$$\frac{d\left(XY^{-1}\right)}{XY^{-1}} = \frac{dX}{X} - \frac{dY}{Y} + \left(\frac{dY}{Y}\right)^2 - \frac{dX}{X}\frac{dY}{Y}.$$

Since $\mathbb{E}_t^O \left(dr_t^{Ok} \right) - r_t = x_t^H \left(\sigma_t^p \right)^2$ from the optimists' optimal portfolio decision à la Merton (1971), and from the fact that

$$\mathbb{E}_t^O \left(dr_t^{Ok} \right) = \mathbb{E}_t \left(dr_t^{Ok} \right) + \frac{\alpha^O - \alpha}{\sigma} \sigma_t^p dt, \tag{B.13}$$

where $\mathbb{E}_t$ is the operator corresponding to the rational expectations, we plug (B.9), (B.12), and (B.13) into (B.10) and obtain

$$
\begin{aligned}
\frac{d\eta_t}{\eta_t} &= \left(\frac{\psi_t - \eta_t}{\eta_t} \sigma_t^p \right)^2 dt - \frac{\psi_t - \eta_t}{\eta_t} \frac{\alpha^O - \alpha}{\sigma} \sigma_t^p dt + \frac{1 - \iota_t^O}{q_t} dt + (1 - \psi_t) \left(\delta^H - \delta^O \right) dt \\
&\quad + (1 - l)(1 - \psi_t) \Lambda^O \left(i_t^O \right) dt - \rho dt + \frac{\psi_t - \eta_t}{\eta_t} \sigma_t^p dZ_t,
\end{aligned}
\tag{B.14}
$$

where we used $c_t^O = \rho W_t^O$ in the equilibrium. From (B.14), we obtain

$$\mu^\eta(\eta_t) = \left(\frac{\psi_t - \eta_t}{\eta_t} \sigma_t^p \right)^2 - \frac{\psi_t - \eta_t}{\eta_t} \frac{\alpha^O - \alpha}{\sigma} \sigma_t^p + \frac{1 - \iota_t^O}{q_t} + (1 - \psi_t) \left(\delta^H - \delta^O \right) + (1-l)(1 - \psi_t) \Lambda^O(i_t^O) - \rho, \tag{B.15}$$

and

$$\sigma^\eta(\eta_t) = \frac{\psi_t - \eta_t}{\eta_t} \sigma_t^p. \tag{B.16}$$

∎

Proof of Proposition 3. Applying Ito's lemma to the Markov relationship $q_t = q(\eta_t)$, we obtain

$$dq_t = q'(\eta_t) d\eta_t + \frac{1}{2} q''(\eta_t)(d\eta_t)^2. \tag{B.17}$$

Plugging in equation (33) to (B.17) and using $(d\eta_t)^2 = \eta_t^2 \sigma^\eta(\eta_t)^2 dt$, we have

$$\frac{dq_t}{q_t} = \left(\eta_t \mu^\eta(\eta_t) \frac{q'(\eta_t)}{q(\eta_t)} + \frac{\eta_t^2 \sigma^\eta(\eta_t)^2}{2} \frac{q''(\eta_t)}{q(\eta_t)} \right) dt + \eta_t \sigma^\eta(\eta_t) \frac{q'(\eta_t)}{q(\eta_t)} dZ_t. \tag{B.18}$$

With the definition of normalized asset price (i.e., price-earnings ratio) $q_t = \frac{p_t}{\gamma_t^O}$, we have:[5]

$$\frac{dq_t}{q_t} = \left(\mu_t^p - \alpha + \sigma^2 - \sigma_t^p \sigma \right) dt + \left(\sigma_t^p - \sigma \right) dZ_t. \tag{B.19}$$

---

[5]Note that in equation (B.19), we use the 'true' process for γ_t^O given in (12).

Comparing (B.18) and (B.19) yields:

$$\sigma_t^p = \sigma + \eta_t \sigma^\eta (\eta_t) \frac{q'(\eta_t)}{q(\eta_t)}, \tag{B.20}$$

and

$$\mu_t^p = \alpha + \eta_t \mu^\eta (\eta_t) \frac{q'(\eta_t)}{q(\eta_t)} + \frac{\eta_t^2 \sigma^\eta (\eta_t)^2}{2} \frac{q''(\eta_t)}{q(\eta_t)} + \sigma \sigma_t^\eta \eta_t \frac{q'(\eta_t)}{q(\eta_t)}. \tag{B.21}$$

From (B.16) in Proposition 2, we have that

$$\sigma^\eta (\eta_t) = \frac{x_t \eta_t - \eta_t}{\eta_t} \sigma_t^p, \tag{B.22}$$

where we used $x_t \eta_t = \psi_t$ from (32). Therefore, by (B.20) and (B.22) we have

$$\sigma_t^p = \frac{\sigma}{1 - (x_t \eta_t - \eta_t) \dfrac{q'(\eta_t)}{q(\eta_t)}}. \tag{B.23}$$

∎

B.1 Price of Capital and the Optimal Leverages

We derive a closed formed solution for leverage multiple and a first-order differential equation for the price of capital. The following summarize the main results of the paper.

Proposition 5 *The equilibrium domain consists of two intervals $[0, \eta^\psi)$, where $\psi(\eta) < 1$, and $[\eta^\psi, 1]$ where $\psi(\eta) = 1$. The capital price $q(\eta)$ in our equilibrium is given by:*

$$q(0) = l \cdot \frac{1 - i(q(0))}{\rho} \tag{B.24}$$

and

$$q(\eta) = \frac{1 - i(q(\eta))}{\rho}, \quad \forall \eta \in [\eta^\psi, 1] \tag{B.25}$$

The following procedures can be used to compute $\psi(\eta)$ and $q'(\eta)$ from $(\eta, q(\eta))$ to $(\eta^\psi, 0)$ when $\eta < \eta^\psi$:

1. *Find ψ that satisfies*

$$\rho q(\eta) = \psi + (1 - \psi)l - i(q)(\psi + (1 - \psi)l) \tag{B.26}$$

2. *Compute $q(\eta)$ where $q(\eta)$ is given by the solution of the equation:*

$$\frac{(1-l)(1-i(q(\eta)))}{q(\eta)}+(1-l)\Lambda^O\left(i(q(\eta))\right)+\delta^H-\delta^O+\frac{\alpha^O-\alpha}{\sigma}\sigma^p(\eta)=\left(\frac{\psi}{\eta}-\frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2,$$
(B.27)

where $\sigma^p(\eta)$ is given by (B.23). From (B.27), $\sigma^p(\eta)$ can also be expressed as:

$$\sigma^p(\eta)=\frac{\frac{\alpha^O-\alpha}{\sigma}+\sqrt{\left(\frac{\alpha^O-\alpha}{\sigma}\right)^2+4\left(\frac{\psi}{\eta}-\frac{1-\psi}{1-\eta}\right)\left(\frac{(1-l)(1-i(q))}{q(\eta)}+(1-l)\Lambda^O(i(q))+\delta^H-\delta^O\right)}}{2\left(\frac{\psi}{\eta}-\frac{1-\psi}{1-\eta}\right)},$$
(B.28)

where $q=q(\eta)$.

Proof of Proposition 5. From the good market equilibrium condition (22), we have

$$k_t^O\left(\gamma_t^O-\iota_t^O\gamma_t^O\right)+k_t^H\left(\gamma^H-\iota_t^H\gamma^H\right)=c_t^O+c_t^H,\quad\forall t\in[0,\infty).$$
(B.29)

Observing that $c_t^O=\rho w_t^O$ and $c_t^H=\rho w_t^H$ in equilibrium, we divide (B.29) by $p_t K_t$ and obtain[6]

$$\rho q=\psi+(1-\psi)l-i(q)\left(\psi+(1-\psi)l\right).$$
(B.30)

Since $\mathbb{E}_t^O\left(dr_t^{Ok}\right)-r_t=x_t^H\left(\sigma_t^p\right)^2$ from the optimists' optimal portfolio decision, we obtain

$$\frac{1-i(q(\eta))}{q(\eta)}+\Lambda^O(i(q(\eta)))-\delta^O+\mu_t^p+\frac{\alpha^O-\alpha}{\sigma}\sigma^p(\eta)-r_t=\left(\frac{\psi}{\eta}\right)\sigma^p(\eta)^2.$$
(B.31)

For households, from Proposition 1, it must be the case where

$$l\cdot\frac{1-i(q(\eta))}{q(\eta)}+l\cdot\Lambda^O(i(q(\eta)))-\delta^H+\mu_t^p-r_t\le\left(\frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2.$$
(B.32)

with equality when $\psi<1$. Finally, by subtracting (B.32) from (B.31), we get

$$\frac{(1-l)(1-i(q(\eta)))}{q(\eta)}+(1-l)\Lambda^O\left(i(q(\eta))\right)+\delta^H-\delta^O+\frac{\alpha^O-\alpha}{\sigma}\sigma^p(\eta)=\left(\frac{\psi}{\eta}-\frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2,$$
(B.33)

when $\psi_t<1$, i.e., households hold some physical capital.

∎

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[6]Since $\iota_t^H(q_t)=\iota_t^O(q_t)$, we use the notation $\iota(q(\eta))$ to denote that $\iota(\cdot)$ is a function of normalized asset price q, which in turn depends on the state variable η.

B.2 Stationary Distribution

We now compute the stationary (i.e., ergodic) distribution of the wealth share of optimists, η_t. Given the η_t process in equation (33), the Kolmogorov forward equation (KFE) is given by

$$0 = -\frac{\partial}{\partial \eta} \left(\mu^\eta(\eta)\eta \cdot d(\eta) \right) + \frac{1}{2} \frac{\partial^2}{\partial \eta^2} \left((\sigma^\eta(\eta)\eta)^2 d(\eta) \right). \tag{B.34}$$

where $d(\eta)$ denotes the distribution. With the transformation $D(\eta) \equiv (\sigma^\eta(\eta)\eta)^2 \cdot d(\eta)$, equation (B.34) can be written as

$$\frac{D'(\eta)}{D(\eta)} = 2\frac{\mu^\eta(\eta)\eta}{(\sigma^\eta(\eta)\eta)^2}, \tag{B.35}$$

which can be solved easily by integrating both sides of (B.35).

Deriving $\mu^\eta(0)$ and $\sigma^\eta(0)$ in (37)   The asymptotic solution of drift $\mu^\eta(\eta)$ and volatility $\sigma^\eta(\eta)$ of η, when $\eta \to 0$ can be computed in a similar way to Brunnermeier and Sannikov (2014). Asymptotically, let us assume that as $\eta \to 0$, $q(\eta) \to q_0$ where q_0 is the equilibrium level of normalized capital price when households hold the entire aggregate wealth. Let as assume $\psi(\eta) \simeq \psi_0 \eta$ when $\eta \sim 0$. Then we know $\sigma_t^p \to \sigma$ (from (35)), $\mu_t^p \to \alpha$ (from (34)), and $\sigma^\eta \to (\psi_0 - 1)\sigma$ thereafter. Following steps are needed to calculate $\mu^\eta(0)$ and $\sigma^\eta(0)$:

1. From (B.24), i.e., $\rho q_0 = l(1 - \iota_o)$ we obtain q_0 and $\iota_0 = \iota(q_0) = \frac{q_0^2 - 1}{2k}$.

2. From (33), we know that as $\eta \to 0$, $\mu^\eta \to \mu_0$, where μ_0 can be written as

$$\mu_0 = ((\psi_0 - 1)\sigma)^2 - (\psi_0 - 1)\frac{\alpha^O - \alpha}{\sigma}\sigma + \frac{1 - \iota_0}{q_0} + (\delta^H - \delta^O) + (1 - l)\Lambda^O(\iota_0) - \rho.$$

3. From (B.27), we know

$$\frac{(1 - l)(1 - \iota_0)}{q_0} + (1 - l)\Lambda^O(\iota_0) + (\delta^H - \delta^O) + \frac{\alpha^O - \alpha}{\sigma}\sigma = (\psi_0 - 1)\sigma^2$$

as

$$\left(\frac{\psi}{\eta} - \frac{1 - \psi}{1 - \eta} \right) \to (\psi_0 - 1).$$

4. Therefore from the above 3 steps, we calculate μ_0 and ψ_0, from which we calculate

$$\sigma^\eta = (\psi_0 - 1)\sigma.$$

$$\mu^\eta(0) \equiv \lim_{\eta \to 0} \mu^\eta(\eta) = \mu_0$$

$$\sigma^\eta(0) \equiv \lim_{\eta \to 0} \sigma^\eta(\eta) = (\psi_0 - 1)\sigma$$

Proof of Corollary 1. From (35), we obtain

$$\lim_{\eta \to 0} \sigma_t^p = \sigma.$$

From L'Hôpital's rule, we also obtain that:

$$\lim_{\eta \to 0} \sigma_t^\eta = \lim_{\eta \to 0} \frac{\psi_t - \eta_t}{\eta_t} \sigma_t^p = \lim_{\eta \to 0} \left(\psi_t'(\eta) - 1 \right) \sigma \equiv \sigma^\eta(0^+).$$

The derivative of ψ increases with optimism as seen in Figure **??**, and, in the limit, the drift μ^η and the volatility σ^η of η do not depend on η. Since now η in the limit (i.e., $\eta \sim 0$) follows a geometric Brownian motion with constant volatility $\sigma^\eta(0^+)$, its skewness is given by:

$$\left(\exp\left((\sigma^\eta(0^+))^2 \right) + 2 \right) \sqrt{\exp\left((\sigma^\eta(0^+))^2 \right) - 1}$$

which is always positive, and increasing in $\sigma^\eta(0^+)$. Therefore, as $\eta \to 0$, the skewness of the stationary distribution $d(\eta)$ increases when optimism $\alpha^O - \alpha$ increases.

∎

B.3 Welfare derivations

From the goods market clearing condition (B.26), i.e., $k_t^O(\gamma^O - \iota^O \gamma^O) + k_t^P(\gamma^P - \iota^P \gamma^P) = C_t$ where C_t is aggregate consumption, and $\frac{C_t}{W_t} = \frac{c_t^H}{w_t^H} = \rho$, we obtain $c_t^h = \rho(1 - \eta_t)W_t = (1 - \eta_t)K_t\gamma_t^O \cdot (1 - \iota_t)A(\psi_t)$. Then we obtain the decomposition of welfare in (39).

For the capital effect, we obtain

$$\underbrace{\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log K_t dt \right]}_{\text{Capital effect}} = \frac{\log K_0}{\rho} + \frac{1}{\rho} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \left(\Lambda^O \left(i_t^O \right) - \delta^O - (1 - \psi_t)(1 - l)\Lambda^O \left(i_t^O \right) \right) dt \right].$$

For the wealth effect, from (33) we obtain

$$d\log(1-\eta_t) = \frac{-d\eta_t}{1-\eta_t} + \frac{1}{2}\frac{(d\eta_t)^2}{(1-\eta_t)^2}$$

$$= \left(-\frac{\eta_t}{1-\eta_t}\mu_t^\eta + \frac{1}{2}\frac{\eta_t^2}{(1-\eta_t)^2}(\sigma_t^\eta)^2\right)dt - \frac{\eta_t}{1-\eta_t}\sigma_t^\eta dZ_t.$$

Therefore,

$$\underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(1-\eta_t)dt\right]}_{\text{Wealth effect}} = \frac{\log(1-\eta_0)}{\rho} + \frac{1}{\rho}\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\left(-\frac{\eta_t}{1-\eta_t}\mu_t^\eta + \frac{1}{2}\left(\frac{\eta_t}{1-\eta_t}\right)^2(\sigma_t^\eta)^2\right)dt\right]$$

where under $\delta^H = \delta^O = 0$, we use

$$\mu_t^\eta = \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p + \frac{1-\iota_t^O}{q_t} + (1-l)(1-\psi_t)\Lambda^O(i_t^O) - \rho,$$

and

$$\sigma_t^\eta = \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p.$$

Investment and misallocation effects can be calculated based on simulations.