Beliefs and the Net Worth Trap

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Abstract

We develop a tractable framework to explore how beliefs about long-term economic growth shape macroeconomic and financial stability. By modeling belief distortions among productive capital users, we provide an analytical characterization of a novel phenomenon termed the "net worth trap", where overly optimistic or pessimistic beliefs of productive agents prevent them from rebuilding wealth, causing permanent inefficiencies. A procyclical swing in beliefs reduces or exacerbates the instability, indicating that the type of belief when the economy is vulnerable has important consequences on financial stability and macroeconomic dynamics.

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Declaration of interests

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1 Introduction

There has been a surge in the literature on macroeconomic models with financial frictions post 2008 financial crisis, aiming to explain crises and high risk premium levels on financial assets. However, most of these models neglect beliefs about the macroeconomic state, with only a few notable exceptions (e.g., Krishnamurthy and Li (2024), Maxted (2023), Camous and Van der Ghote (2023)). We aim to bridge this gap by investigating how the beliefs of market participants, especially institutional investors or financial intermediaries,[1] about the long-term macroeconomic state of the economy influence financial stability and aggregate macroeconomic dynamics.

This paper contributes in two major ways. First, we introduce a tractable model with two types of agents who differ in their beliefs about macroeconomic growth. One type, who is more productive in capital utilization, has dogmatic beliefs about the long-run growth rate of the economy, which contrasts with the other type, who is less productive but has rational expectations.[2] The beliefs of more productive agents generate expectation errors about their capital returns, leading to financial instability at the aggregate level. Second, we show that beyond a certain threshold level of beliefs, forecast errors arising from distortions generate a "net worth trap", a phenomenon that concentrates capital exclusively in the hands of less productive agents and perpetuates long-term inefficiencies. In addition, belief distortions negatively impact the welfare of rational agents, primarily by generating a net worth trap where capital prices and the economy's growth rates are depressed for a long period.

We purposely build a model with a strong emphasis on financial frictions and risk premia following He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), Gertler et al. (2020), etc., to study implications on financial and macroeconomic stability as well as on capital returns. In these previous works, the aggregate net worth of leveraged agents is a natural measure of financial stability. The main addition in our framework is a belief distortion of agents, which leads to extreme behaviors of their net worth.[3]. The first main insight that emerges from the model is that large belief distortions exacerbate financial instability,

[1] DeVault et al. (2019), for example, find that commonly used sentiment measures capture demand shocks of not household (retail) investors but institutional investors. Likewise, in this paper, we focus on cases where financial intermediaries, i.e., more productive agents, have distorted beliefs.

[2] This assumption aligns with the literature that portrays entrepreneurs as skilled and optimistic. See, e.g., Coval and Thakor (2005). Following the literature, for example, Brunnermeier and Sannikov (2014), we call more (less) productive agents 'experts' ('households').

[3] We focus on cases where belief distortion is on the more productive users of capital. This partly reflects the empirical evidence found, e.g., in Puri and Robinson (2007)

similar to Camous and Van der Ghote (2023).[4]

Under optimistic beliefs, experts expect a higher long-run growth rate of the economy than the true growth rate, and these distorted expectations influence their portfolio choice over risky capital. A series of negative shocks in the economy creates two countervailing effects: (i) a positive effect that aids optimists in rebuilding their net worth through high risk premium, and (ii) a negative effect stemming from their expectation errors counteracting the net worth rebuilding process. The second effect arises because the optimists take their portfolio decisions based on the perceived growth rate which is higher than the true growth rate of the economy. For lower levels of optimism, the first effect dominates, helping the experts to recapitalize and leading to a non-zero probability that the economy moves toward normalcy where all capital is allocated to experts.

However, beyond a certain threshold level of optimism, the second effect dominates, wiping out the experts' wealth and allocating all capital to less productive agents indefinitely. We call this novel phenomenon a *net worth trap*, under which experts remain unable to rebuild their wealth permanently. When experts show pessimism, a similar mechanism is at play that generates a net worth trap. For moderate levels of pessimism, experts reduce their capital holdings but are still leveraged as they are more productive. This enables them to grow their net worth by earning the risk premium. Once pessimism crosses a threshold, their portfolio holdings relative to rational agents become lower. This implies that rational agents take over the economy in the long run by earning the high risk premium. In other words, the share in risky capital is too low for the experts to grow their wealth.

A benchmark economy with rational expectations does not feature a net worth trap. This is because the risk premium effect is the only working channel in the absence of belief distortion. In the stochastic steady state, experts earn a large risk premium that helps them continuously grow their wealth, eventually taking over the economy.[5] While models with financial frictions and belief distortions such as Krishnamurthy and Li (2024) and Maxted (2023) feature a fat tailed stationary distribution, we offer an analytical characterization of an extreme case of the instability where the stationary distribution endogenously becomes

[4]Camous and Van der Ghote (2023) build a model with diagnostic expectations where risk-neutral agents face binding leverage constraints and capital-finance providers do not consume. In our model, all agents are risk-averse with preferences over consumption, and optimists face a skin-in-the-game constraint that makes the market incomplete.

[5]This is a common feature in many macro-finance models, e.g., Krishnamurthy and Li (2024), He et al. (2017). In particular, Gopalakrishna (2022) avoids this problem by introducing an exit rate for experts. With an exit rate, the economy will have a non-degenerate stationary density, meaning that there will be no net worth trap.

a Dirac-delta measure at the most inefficient region.[6] Our net worth trap provides a belief-based explanation to a slow-moving capital from crisis, in line with Duffie (2010).

A surprising result we derive is that beliefs of agents about the growth rate in the limit where their net worth approaches zero is the primary driver of net worth trap, meaning that beliefs in the stochastic steady state is not important at all. To demonstrate this, we analyze a case where experts are optimistic in the stochastic steady state but become pessimistic in a crisis, where their net worth is close to zero. Since crisis itself is endogenous, this type of belief formation captures an endogenous swing in their sentiment and has a similar flavor to diagnostic expectations (see, for example, Bordalo et al. (2018), and Maxted (2023)), in the sense that experts irrationally revise down (up) the growth rate in bad (good) states of the economy. Whether a net worth trap arises under this type of belief formation depends on the extent of pessimism when the wealth of experts is close to zero. For a low level of pessimism, the risk premium channel dominates the expectation error channel, and thus acts as a stabilizing force, helping the experts to rebuild wealth. For higher levels of pessimism, experts reduce more their portfolio holding of risky capital, helping the rational agents earn risk premium and take over the economy in the long run, leading to permanent inefficiency. This result is without loss of generality with respect to beliefs at the stochastic steady state both in terms of direction (optimism or pessimism) and levels. Only beliefs in the limit as net worth of experts approaches zero matters for the long run survival of experts.

The elicitation of objectively accurate beliefs about the underlying technological process (i.e., rational expectations) is out of the scope of this paper. In most parts, experts in our framework have static and dogmatic beliefs about their expected long-run growth rate. In this formulation, the belief is persistent and agents never learn from the data. This setting is in accordance with the literature where variation about individual beliefs about expected returns are due to individual fixed effects (e.g., Giglio et al. (2019)). In contrast to the models of e.g., Harrison and Kreps (1978), Scheinkman and Xiong (2003), and Geanakoplos (2010), we consider risk-averse, not risk-neutral agents. On the methodological side, we normalize the economy by technology so that our economy becomes stationary, and rely on the Kolmogorov forward equation (KFE) in characterizing the ergodic distribution of our state variable, in order to illustrate the long term dynamics and especially, the net worth trap.

[6] In fact, our model does not have any additional assumptions such as portfolio and collateral constraints.

Related literature Our paper relates to a large literature on macro-finance models with financial frictions playing a central role in amplifying shocks (e.g., He and Krishnamurthy (2013), Gertler et al. (2020) Brunnermeier and Sannikov (2014)). Recent works in this literature have turned to quantitative models that explain macroeconomic and asset pricing moments either using a rational expectations model, or using a belief based model.[7] We follow this line of literature but focus on the general equilibrium interactions between the endogenous risk and macroeconomic variables in the presence of optimism about the long-run growth. Specifically, we build a tractable model with dogmatic beliefs that generates a net worth trap perpetuating financial instability in the long run. There is a large literature that focuses on belief heterogeneity (see, for example, Harrison and Kreps (1978), Simsek (2013), and Caballero and Simsek (2020)). Harrison and Kreps (1978) assume that agents agree to disagree about their beliefs, and asset prices can exceed their fundamental values in that case. Simsek (2013) studies cases in which optimists borrow from pessimists using loans collateralized by the asset that optimists purchase. As pessimists attach lower values to the collateral, this collateral arrangement puts the endogenous borrowing constraint on optimists, affecting their leverage choices and asset prices. Brunnermeier (2024) relates the net worth trap to economic resilience, making a distinction between a trap with escape and without escape. The net worth trap in this paper denotes the case without escape.

We also relate to a long literature on deviations from the rational expectations, e.g., Detemple and Murthy (1997), Basak and Croitoru (2000), Basak (2000), Basak and Croitoru (2006), Gallmeyer and Hollifield (2008), Chabakauri (2015), and Dong et al. (2022).[8] The difference is that we focus on the implications on financial stability and the interaction between macroeconomy and asset prices. In this regard, we are similar to Maxted (2023) and Camous and Van der Ghote (2023). Camous and Van der Ghote (2023) introduces diagnostic expectations into a canonical macro-finance model with binding leverage constraints and shows that diagnostic expectations exacerbate financial instability. On the contrary, the diagnostic expectation decreases financial instability compared to a rational expectations benchmark in Maxted (2023). Our financial stability results based on swinging beliefs can generate both results, depending on the degrees of optimism and pessimism. In addition,

[7] Maxted (2023) builds a macro-finance model with diagnostic expectations, and Krishnamurthy and Li (2024) builds a model where agents update their beliefs about tail risk rationally. In a recent work, Gopalakrishna (2022) introduces state-dependent exits of experts into a canonical rational expectations model to generate amplified (in volatility and risk-premium) but 'slow-moving' financial crises.

[8] In Dong et al. (2022), optimistic investors buy risky assets with leverage provided by pessimists, pushing up asset prices like in our model. Higher asset prices relax the financial frictions imposed on high productivity firms, mitigating degrees of misallocation and raising aggregate output.

we characterize an extreme case of instability featuring a net worth trap where the stationary distribution has a sharp peak around the zero net worth of experts. Our assumption of beliefs about long-term growth rate of the economy relates to Bordalo et al. (2023) who find that analysts' optimism about long-term earnings growth of S&P500 firms amplifies the boom-bust cycle and explains the excess volatility of the business cycle. This finding is in line with our result that optimism about long-term technological growth increase the aggregate macroeconomic instability.

Finally, this paper relates to the empirical literature focusing on heterogeneous beliefs across different groups of market agents. For example, Welch (2000) finds there is a large degree of heterogeneity in forecasted risk premium levels even across financial economists, while Beutel and Weber (2022) point out individuals are heterogeneous both at the information acquisition and the processing stages, thereby forming their own beliefs and choosing portfolios based on those beliefs.[9] Our results are based on the *perceived* equity premium levels across two groups (i.e., optimists and households), providing novel implication about the interaction between optimistic belief and the crisis dynamics in general equilibrium.

Outline The remainder of the paper is organised as follows: Section 2 sets out the basic model. In particular, Section 2.5 presents an analytical theory of net worth trap. Section 3 concludes. Appendix A presents additional figures, and Appendix B provides proofs.

Online Appendices C and D show that our result that a net worth trap exists under extreme behavioral biases is robust across different model specifications under both complete and incomplete markets.

2 The Model

We develop a continuous-time framework with two types of agents: experts and households, based on which we will study how beliefs about technological growth affect leverage choices, asset prices and endogenous financial volatility. First, we assume that experts have

[9] The literature also points out that different groups in the economy (e.g., households, firms, professional forecasters, etc) form different expectations not just about risk premium but also about macroeconomic variables: see e.g., Coibion et al. (2020), Candia et al. (2021), and Weber et al. (2022). As risk premium depends on the business cycle (e.g., Cooper and Priestley (2009)), we can expect that the forecasted risk-premium levels across groups would differ. For equity premium, Rapach et al. (2012) uncover that despite the failure of *individual* out of sample forecasts to outperform the historical average, *combinations* of individual forecasts deliver significant out-of-sample gains relative to the historical average on a consistent basis over time.

dogmatic beliefs about technological growth, and later relax this assumption to incorporate a swinging sentiment that depends on macroeconomic environments.

The basic model is built on e.g., Basak (2000) and Brunnermeier and Sannikov (2014), is analytically tractable, and incorporates exogenous technological growth and heterogeneous beliefs.

2.1 Model Setup

We begin with the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ which is endowed with a standard Brownian motion Z_t. We assume that $Z_0 = 0$, almost surely. All economic activity will be assumed to take place in the horizon $[0, \infty)$. Let

$$\mathcal{F}^Z(t) = \sigma\{Z_s;\ 0 \leq s \leq t\}, \forall t \in [0, \infty)$$

be the filtration generated by $\{Z_s\}_{s \leq t}$ and let $\mathcal{N}$ denote the $\mathcal{P}$- null subsets of $\mathcal{F}^Z(\infty)$. We shall use the augmented filtration as follows:

$$\mathcal{F}(t) = \sigma\left\{\mathcal{F}^Z(t) \cup \mathcal{N}\right\}, \forall t \in [0, \infty).$$

One should interpret the $\sigma-$algebra $\mathcal{F}(t)$ as the information available to agents at time t in a complete information setting, in the sense that if $\omega \in \Omega$ is the true state of nature and if $A \in \mathcal{F}(t)$, then all agents will know whether $\omega \in A$.

We consider an economy filled with two types of agents, experts and households. Both types can own capital, but the former are able to employ capital in a more productive way. Experts potentially have a distorted belief, which we elaborate later.[10]

2.1.1 Technology

The aggregate amount of capital in the economy is denoted by K_t and capital owned by an individual agent i by k_t^i , where $t \in [0, \infty)$ indicates time. The physical capital k_t^O held by experts produces output at rate:

$$y_t^O = \gamma_t^O k_t^O, \quad \forall t \in [0, \infty) \tag{1}$$

[10]Our assumption that more productive experts have distorted beliefs is from the literature, e.g., Coval and Thakor (2005) and Chabakauri (2015).

per unit of time, where γ_t^O is an exogenous productivity parameter, which evolves according to:[11]

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha dt + \sigma dZ_t, \quad \forall t \in [0, \infty), \tag{2}$$

where dZ_t is the aggregate standard Brownian motion defined above. The output is modeled as a numeraire, and therefore, its price is normalized to one. Capital owned by individual optimists satisfies the following Ito's process:

$$\frac{dk_t^O}{k_t^O} = \left(\Lambda^O(\iota_t^O) - \delta^O\right) dt, \quad \forall t \in [0, \infty), \tag{3}$$

where ι_t^O is the portion of the generated output (i.e., $y_t^O = \gamma_t^O k_t^O$) used in creating new capital (i.e., the investment made during an infinitesimal period $(t, t+dt)$ is $\iota_t^O \gamma_t^O k_t^O dt$). In (3), function $\Lambda^O(\cdot)$, which satisfies $\Lambda^O(0) = 0$, $\Lambda^{O\prime}(0) = 1$, $\Lambda^{O\prime}(\cdot) > 0$, and $\Lambda^{O\prime\prime}(\cdot) < 0$, represents a standard investment technology with adjustment costs. In cases where there is no investment, the capital managed by experts depreciates at a rate δ^O.[12]

Households are less productive users of capital. The capital k_t^H managed by households produces the following output:

$$y_t^H = \gamma_t^H k_t^H, \quad \forall t \in [0, \infty), \tag{4}$$

with γ_t^H is given by $\gamma_t^H = l \cdot \gamma_t^O \leq \gamma^O$, where $l \leq 1$, and evolves according to:

$$\frac{d\gamma_t^H}{\gamma_t^H} = \frac{d\gamma_t^O}{\gamma_t^O}, \quad \forall t \in [0, \infty). \tag{5}$$

In other words, experts have a proportionally higher productivity than households, where the proportionality is given by $l \leq 1$, and productivity of two groups of agents grows at the same rate at every instant. The capital owned by households follows:

$$\frac{dk_t^H}{k_t^H} = \left(\Lambda^H(\iota_t^H) - \delta^H\right) dt, \quad \forall t \in [0, \infty), \tag{6}$$

where in the same way as above, ι_t^H is a capital-owning household's investment rate per

[11] Therefore, unlike Brunnermeier and Sannikov (2014), we have the stochastic growth which we model as exogenous (i.e., α and σ in (2) are exogenous). Later we normalize our economy by γ_t^O to make it stationary.

[12] In contrast to the literature (e.g., Brunnermeier and Sannikov (2014)), we abstract from the capital risk usually assumed. Its inclusion does not change our results qualitatively, as seen in Online Appendices C and D.

unit of output. The state space $\mathscr{F}^k$ satisfies the same conditions as for experts and $\delta^H \geq 0$ is the depreciation rate when the capital is managed by households. We assume that $\Lambda^H(\iota) = l \cdot \Lambda^O(\iota)$ for $\forall \iota$ with $l < 1$ for simplicity.[13]

2.1.2 Preferences

Experts and households have the same preferences that are generally characterized by the instantaneous utility function $u(c_t^i) : \mathbb{R}_+ \to \mathbb{R}$, where $i \in \{O, H\}$, with the same discount rate ρ. The consumption space defined above must also be square-integrable:

$$\int_0^\infty \left|c_t^i\right|^2 dt < \infty.$$

With $c^i \equiv \{c_t^i\}_{t=0}^\infty$, each agent of type $i \in \{O, H\}$ maximizes her expected lifetime utility given by:

$$U(c^i) = \mathbb{E}_0^i \int_0^\infty e^{-\rho t} u\left(c_t^i\right) dt, \quad \forall t \in [0, \infty). \tag{7}$$

where $\mathbb{E}_0^i$ is the expectations operator of type $i \in \{O, H\}$. The utility function obeys the standard assumption that $u(c_t^i)$ is continuously differentiable, increasing, and concave in c_t^i.

2.1.3 Market for Capital

Both experts and households have the opportunity to trade the physical capital in a competitive market. We denote the equilibrium market price of capital in terms of the output by p_t, and assume it satisfies the following endogenous process:

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p dZ_t, \tag{8}$$

where μ_t^p and σ_t^p are drift and volatility of the capital price process (8), respectively. Based on the definition, capital k_t^O costs $p_t k_t^O$ for experts. Note that, in equilibrium, p_t, μ_t^p and σ_t^p are determined endogenously.

[13] Thus, we effectively assume that both (i) households' productivity in turning capital into output; and (ii) their productivity in turning output into capital are both lower than of experts with the same proportionality $l \leq 1$. This is for tractability.

2.1.4 Return to Capital

A straightforward application of Ito's lemma with (3) and (8) reveals that when an expert holds k_t^O units of capital, the total value of this capital $p_t k_t^O$ evolves according to:

$$\frac{d(p_t k_t^O)}{p_t k_t^O} = \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t. \tag{9}$$

Hence, the total return that experts earn from capital (per unit of wealth invested) is given by:

$$dr_t^{Ok} = \frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t. \tag{10}$$

Similarly, a household earns the return

$$dr_t^{Hk} = \frac{\gamma_t^H - \iota_t^H \gamma_t^H}{p_t} dt + \left(\Lambda^H(\iota_t^H) - \delta^H + \mu_t^p\right) dt + \sigma_t^p dZ_t. \tag{11}$$

2.1.5 Beliefs and Aggregate Consistency

Rational households The households are rational, i.e., they know that their productivity γ_t^H follows the process in (2) and (5). They observe realized dZ_t in each period, and have a full knowledge of the above aggregate processes, i.e., equations (8), (9), (10), and (11).

Experts Experts, in contrast, observe their productivity γ_t^O at any instant, but have incomplete information on its exact dynamics. With equation (2), γ_t^O actually follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha dt + \sigma dZ_t, \quad \forall t \in [0, \infty). \tag{12}$$

While experts observe the left-hand side, $\frac{d\gamma_t^O}{\gamma_t^O}$, they do not observe realized dZ_t, an actual Brownian motion, and do not know true α. Instead, they believe that their productivity γ_t^O follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha^O dt + \sigma dZ_t^O, \quad \forall t \in [0, \infty), \tag{13}$$

where α^O is the perceived growth rate possibly different from α, and Z_t^O is their perceived Brownian motion. That is, the probability triplet that experts have is $(\Omega, \mathcal{F}^0, \mathcal{P}^O)$, where $\mathcal{F}^0$ is the filtration observed by the experts and $\mathcal{P}^0$ is their subjective probability measure under which Z_t^0 is a $(\mathcal{F}^0)$-Brownian motion. With $\alpha^O \neq \alpha$, the experts believe that their productivity's growth rate is different from its true rate α in (2). Experts are dogmatic in their beliefs and do not learn from the realized data, in a similar manner to Yan (2008) and Chabakauri (2015).[14] We call experts optimists (pessimists) when $\alpha^O > \alpha$ ($\alpha^O < \alpha$).

Aggregate consistency From equations (12), and (13), we obtain the following aggregate consistency condition for optimists:

$$Z_t^O = Z_t - \frac{\alpha^O - \alpha}{\sigma} t. \tag{14}$$

In other words, experts regard Z_t^O, not Z_t, as the true Brownian motion, while Z_t^O is not a Brownian motion under the rational expectations. This condition acts as an additional equilibrium condition in our model. The aggregate capital price under the experts' measure follows:

$$\frac{dp_t}{p_t} = \mu_t^{p,O} dt + \sigma_t^p dZ_t^O. \tag{15}$$

where the relation between $\mu_t^{p,O}$ and μ_t^p is given by

$$\mu_t^{p,O} = \mu_t^p + \frac{\alpha^O - \alpha}{\sigma} \sigma_t^p, \tag{16}$$

Experts believe the growth rate of price p_t to be $\mu_t^{p,O}$, not μ_t^p, where both are determined in equilibrium. The price volatility σ_t^p is the same under actual measure $\mathcal{P}$ and experts' subjective measure $\mathcal{P}^O$. This can be seen from substituting (14) in the actual price process (8). From equations (10) and (14), experts believe that the total return that they earn from capital holding (per unit of wealth invested) follows

$$dr_t^{Ok} = \left[\frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^{p,O} \right) \right] dt + \sigma_t^p dZ_t^O. \tag{17}$$

Two points regarding the return process in (17) are worth noticing: (i) with $\alpha^O \neq \alpha$, experts believe that the expected capital gain they earn when investing in capital is different from the one implied under the rational expectations; (ii) the degree of beliefs in terms of the 'expected' return (i.e., $\frac{\alpha^O - \alpha}{\sigma} \sigma_t^p$) becomes proportional to the endogenous capital price volatility σ_t^p: a higher endogenous risk σ_t^p mechanically raises the impact of beliefs in asset

[14] This assumption of dogmatic beliefs can be restrictive but allows us to get analytical characterizations of the model. Later in Section 2.5.2 we relax this assumption to incorporate a swinging sentiment of experts that depends on macroeconomic environments.

returns.

2.2 Consumption-Portfolio Problems

Experts Experts can invest in the physical capital, or the risk free asset, which is in zero net supply. w_t^O, the net worth of expert who invests fraction x_t^O of her wealth in capital and consumes with the rate c_t^O, evolves according to:

$$dw_t^O = x_t^O w_t^O dr_t^{Ok} + (1 - x_t^O) r_t w_t^O dt - c_t^O dt, \tag{18}$$

where r_t is the risk-free interest rate prevailing in the economy. Note that r_t is an equilibrium object to be determined endogenously. Formally, each expert solves

$$\max_{x_t^O \geq 0, c_t^O \geq 0, \iota_t^O} \mathbb{E}_0^O \left[\int_0^\infty e^{-\rho t} u\left(c_t^O\right) dt \right], \tag{19}$$

subject to solvency $w_t^O \geq 0$ and the dynamic budget constraint (18). In optimization (19), the expectation operator $\mathbb{E}_0^O$ means that experts believe dZ_t^O, not dZ_t, is the true Brownian motion. Therefore in characterizing (18), they use (17) for the capital return process dr_t^{Ok} instead of (10) with the true dZ_t. No shorting constraint implies $x_t^O \geq 0$.

Households In a similar way to experts' problem in (19), the net worth w_t^H of households, who invest fraction x_t^H of their wealth in capital and consume with rate c_t^H, would follow

$$dw_t^H = x_t^H w_t^H dr_t^{Hk} + w_t^H (1 - x_t^H) r_t dt - c_t^H dt. \tag{20}$$

Formally, each household solves

$$\max_{x_t^H \geq 0, c_t^H \geq 0, \iota_t^H} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} u\left(c_t^H\right) dt \right], \tag{21}$$

subject to the solvency constraint $w_t^H \geq 0$ and the dynamic budget constraints (20). Unlike experts, in optimization (21), the expectation operator $\mathbb{E}_0$ means that households have the rational expectations: they believe dZ_t is the true Brownian motion. Therefore in characterizing (20), they use (11) for the capital return process dr_t^{Hk} with the true dZ_t. Note that we assume a no shorting constraint for capital since we interpret it as physical capital following Brunnermeier and Sannikov (2014). Hence $x_t^j \geq 0, j \in \{O, H\}$. This assumption is not crucial for our key results in Section 2.5.

2.3 Equilibrium and Market Clearing

Equilibrium with full information is characterized by a map from shock histories $\{Z_S, s \in [0,t]\}$, to the prices p_t and asset allocations such that, given prices, agents maximize their expected utilities[15] and markets clear.[16] Since experts deviate from the rational expectations with their own perceived shocks Z_t^O, they have their own expectation operator $\mathbb{E}_0^O$ as shown in (19). The market clearing conditions are as follows

- Capital market

$$\int_0^1 k_t^O di + \int_1^2 k_t^H dj = K_t, \ \ \forall t \in [0, \infty), \tag{22}$$

- Goods market[17]

$$\int_0^1 k_t^O \left(\gamma_t^O - \iota_t^O \gamma_t^O\right) di + \int_1^2 k_t^H \left(\gamma_t^H - \iota_t^H \gamma_t^H\right) dj = \int_0^1 c_t^O di + \int_1^2 c_t^H dj, \ \ \forall t \in [0, \infty). \tag{23}$$

The aggregate capital K_t follows

$$dK_t = \left(\int_0^1 \left(\Lambda^O\left(\iota_t^O\right) - \delta^O\right) k_t^O di + \int_1^2 \left(\Lambda^H\left(\iota_t^H\right) - \delta^H\right) \underline{k}_t^H dj\right) dt, \ \ \forall t \in [0, \infty). \tag{24}$$

Definition 1 *The equilibrium consists of the stochastic processes of (i) the price of capital p_t, (ii) risk free rate r_t, (iii) investment and consumption rates, i.e. $\left\{\left(k_t^O, \iota_t^O, c_t^O\right)\right\}_{t \geq 0}$ for experts, and $\left\{\left(k_t^H, \iota_t^H, c_t^H\right)\right\}_{t \geq 0}$ for households, satisfying the following three conditions:*

1. Given their perceived Brownian motion Z_t^O, experts $O \in [0,1]$ solve optimization (19) subject to their dynamic budget constraint (18) and the solvency condition $w_t^O \geq 0$. Households $H \in (1,2]$ solve optimization (21) subject to their budget constraint (20) and solvency $w_t^j \geq 0$, under rational expectations.

2. The capital market (i.e., (22) and (24)), goods market (i.e., (23)), and debt market are cleared.

[15] Experts solve optimization (19) subject to their dynamic budget constraint (18) and the solvency $w_t^O \geq 0$ while the households solve optimization (21) subject to their dynamic budget constraint (20) and the solvency $w_t^H \geq 0$.

[16] The physical capital, output, and debt markets must clear in equilibrium.

[17] Debt market trivially clears by Walras' law.

3. The consistency condition (14) between Z_t^O and Z_t is satisfied.

2.4 Equilibrium Characterization

We derive a Markov equilibrium in terms of the wealth share of experts defined as

$$\eta_t = \frac{W_t^O}{W_t^O + W_t^H} = \frac{W_t^O}{p_t K_t}. \tag{25}$$

where W_t^O and W_t^H are aggregate wealth of experts and households, respectively. We focus on aggregate wealth because all agents within their respective group are identical. We have

$$x_t^O \leq \frac{1}{\eta_t}, \tag{26}$$

which translates to that the maximum 'leverage' that experts can obtain is bounded above by $\frac{1}{\eta_t}$. We identify two regions: the first is when the inequality (26) binds and the second is when leverage is strictly less than $\frac{1}{\eta_t}$, i.e., when the inequality (26) does not bind. We call the first region *normal* and the second region by *crisis*. In other words, in the *normal* region, all physical capital is owned by experts, while in the *crisis* regime, some capital is purchased by households.

We assume the investment function of the experts to be $\Lambda^O(\iota_t^O) = \frac{1}{k}\left(\sqrt{1+2k\iota_t^O} - 1\right)$, which satisfies all the standard assumptions in Section 2.1.1. From now on, we express our equilibrium with the following normalized asset price $q_t \equiv \frac{p_t}{\gamma_t^O}$. The normalized asset price q_t can also be interpreted as the price-earnings ratio of the physical capital of experts.

2.4.1 Solving the Consumption-Portfolio Problems

The expert solves the optimization (19) subject to the dynamics of her wealth (18) and the solvency $w_t^O \geq 0$. Likewise, each household optimizes her lifetime utility (21) subject to the evolution of his wealth (20) and the solvency $w_t^H \geq 0$. We assume that both experts and households have the logarithmic utility function, i.e. $u(c_t^O) = \log c_t^O$, $u(c_t^H) = \log c_t^H$ for mathematical tractability.[18]

Proposition 1 *The optimal policies are as follows.*

[18]The log-utility assumption is relaxed with the introduction of Duffie-Epstein-Zin type stochastic differential utility in Appendix B.3 and Online Appendices C and D.

1. *The optimal consumption c_t^{O*} is given by $c_t^{O*} = \rho w_t^O$. The household's consumption c_t^{H*} is given by $c_t^{H*} = \rho w_t^H$.*

2. *The equilibrium interest rate r_t^* is given by:*

$$r_t^* = \left(\frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p\right) - x_t^O \left(\sigma_t^p\right)^2,$$

where x_t^O is each optimist's optimal portfolio choice (i.e., leverage multiple) as defined in (18). Given this r_t^*, the optimal portfolio choice of households x_t^H as defined in (20) is given by:[19]

$$x_t^H = \max\left\{\frac{\left(\frac{\gamma_t^H - \iota_t^H \gamma_t^H}{p_t} + \Lambda^H(\iota_t^H) - \delta^H + \mu_t^p\right) - r_t^*}{\left(\sigma_t^p\right)^2}, 0\right\}.$$

3. *The optimal investment rates ι_t^{O*} and ι_t^{H*} are given by:*

$$\iota_t^{O*}(q_t) = \iota_t^{H*}(q_t) = \frac{q_t^2 - 1}{2k}.$$

Note that given ι_t^O, μ_t^p, σ_t^p, and x_t, the equilibrium interest rate r_t^* is increasing in beliefs $\alpha^O - \alpha$. If experts believe that the expected capital gain they earn when investing in capital is greater, they will borrow more from households to increase leverage, raising the equilibrium interest rate r_t^*. It is easier to denote leverage of experts and households in terms of capital share defined as $x_t^O = \frac{\psi_t}{\eta_t}$ and $x_t^H = \frac{1-\psi_t}{1-\eta_t}$, where the capital share of experts is defined as $\psi_t = \frac{k_t^O}{K_t}$. The dynamics of the state variable is as follows.

Proposition 2 *The evolution of optimists' wealth relative to the entire economy is given by*

$$\frac{d\eta_t}{\eta_t} = \mu^\eta(\eta_t)dt + \sigma^\eta(\eta_t)dZ_t \tag{27}$$

where

$$\mu^\eta(\eta_t) = \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p + \frac{1 - \iota_t^O}{q_t} + (1 - \psi_t)\left(\delta^H - \delta^O\right) + (1-l)\left(1 - \psi_t\right)\Lambda^O(\iota_t^O) - \rho,$$

[19] Therefore, if the households' *perceived* risk-premium levels are below 0, they only invest in risk-free loans issued by optimists.

and

$$\sigma^{\eta}(\eta_t) = \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p.$$

We can observe from equation (27) that optimism (i.e., $\alpha^O > \alpha$) given $\psi_t \geq \eta_t$ (which holds in the efficient region[20]) lowers the drift $\mu^{\eta}(\eta_t)$. It implies that the optimism hinders the process of optimists' wealth recapitalization and tends to lower their wealth share in the long run, potentially generating a 'net worth trap' when the level of optimism, i.e., $\alpha^O - \alpha$, is high enough. This is caused by the optimists' expectation errors (i.e., wrong judgment about the capital return), and our main focus in Section 2.5. We derive a closed-formed solution in our Markov equilibrium for leverage multiple and a first-order differential equation for the price of capital in Appendix B.1.

Calibration In the following sections, we will use the following parameter calibration when solving our model numerically. $\alpha^O = \alpha = 0.02$ case corresponds to the case where both experts and households are rational, i.e., $\mathbb{E}_t^O = \mathbb{E}_t$.

	Parameter Description	Value	Source (target)
ρ	Discount rate	0.03	Standard: e.g., Brunnermeier and Sannikov (2014).
α	Productivity growth.	0.02	2% growth in the long run.
σ	Exogenous TFP volatility	0.0256	Schmitt-Grohé and Uribe (2007)
δ	Depreciation rate (δ^H, δ^O)	0	2% capital growth in the long run (2.5% in the model's stochastic steady state).
k	Investment function	851.6	Consumption-to-output ratio at 69%
l	Productivity gap	0.7	Most severe recessions: the average output drop from the trend in the Great Depression was $\sim 30\%$ according to Romer (1993).

Table 1: Calibrated parameters

2.5 Net Worth Trap

We characterize the stationary distribution $d(\eta_t)$ of expert's wealth share η_t and study its limit $\eta_t \to 0^+$ to explain the net worth trap, a novel phenomenon that leads to experts losing

[20]In the efficient region, all the capital is held by optimists, thereby $\psi_t = 1$.

all their wealth in the long run. Given η_t process in (27), the Kolmogorov forward equation (KFE) is given by

$$0 = -\frac{\partial}{\partial \eta}\left(\mu^\eta(\eta)\eta \cdot d(\eta)\right) + \frac{1}{2}\frac{\partial^2}{\partial \eta^2}\left(\left(\sigma^\eta(\eta)\eta\right)^2 d(\eta)\right). \tag{28}$$

where $d(\eta)$ denotes the distribution. With the transformation $D(\eta) \equiv \left(\sigma^\eta(\eta)\eta\right)^2 \cdot d(\eta)$, (28) can be written as

$$\frac{D'(\eta)}{D(\eta)} = 2\frac{\mu^\eta(\eta)\eta}{\left(\sigma^\eta(\eta)\eta\right)^2}, \tag{29}$$

which can be solved easily by integrating both sides of the equation.

Lemma 1 *In the limit $\eta \to 0^+$, the drift $\mu^\eta(0^+)$ and diffusion $\sigma^\eta(0^+)$ of the wealth share of experts is given by*

$$\mu^\eta(0^+) \equiv \lim_{\eta \to 0}\mu^\eta(\eta) = \Gamma_0(\alpha^O - \alpha) + \Gamma_0^2\sigma^2 + \Delta_0$$
$$\sigma^\eta(0^+) \equiv \lim_{\eta \to 0}\sigma^\eta(\eta) = \frac{\alpha^O - \alpha}{\sigma} + \Gamma_0\sigma.$$

where

$$\Gamma_0 = \frac{1}{\sigma^2}\left[(1-l)\frac{1-\iota_0}{q_0} + (\delta^H - \delta^O) + (1-l)\Lambda^O(\iota_0)\right]$$
$$\Delta_0 = \frac{1-\iota_0}{q_0} + (\delta^H - \delta^O) + (1-l)\Lambda^O(\iota_0) - \rho$$

and the quantities $\iota_0 = \lim_{\eta \to 0}\iota(\eta)$ and $q_0 = \lim_{\eta \to 0}q(\eta)$ are given in Appendix B.2.

Lemma 1 presents the drift and volatility of η_t in the limit as it approaches zero. The key insight is that belief distortions affect drift and volatility differently and therefore govern the ability of experts to escape the trap point $\eta = 0$. For large belief distortions, endogenous volatility becomes high enough and prohibits the economy to escape form the singularity point of zero net worth. We formalize this in the following key proposition of the paper by highlighting how large belief distortions should be to generate a net worth trap.

Proposition 3 (Net Worth Trap) *There exists a threshold level of belief beyond which the economy is trapped at $\eta = 0$, and the probability of recapitalization for experts converges to zero. For the optimistic case, i.e., $\alpha^O > \alpha$, the threshold is determined by*

$$\alpha^O - \alpha > \sigma\sqrt{\Gamma_0^2\sigma^2 + 2\Delta_0}, \tag{30}$$

and for the pessimistic case, i.e., $\alpha^O < \alpha$, the threshold is given by

$$\alpha^O - \alpha < -\min\left\{\sigma\sqrt{\Gamma_0^2\sigma^2 + 2\Delta_0}, \max\left\{\sigma^2\left(1+\Gamma_0\right), \Delta_0 + \frac{1}{2}\sigma^2\right\}\right\}. \tag{31}$$

Proof. We analyze the tail of the stationary density to pin down a threshold value of belief that generates a net worth trap. From (29) in Appendix B.2, the asymptotic solution for the stationary density $d(\eta)$ when $\eta \sim 0$ is given by

$$d(\eta) \sim \left(\frac{2\mu^\eta(0)}{(\sigma^\eta)^2(0)} - 1\right)\eta^{\frac{2\mu^\eta(0)}{(\sigma^\eta)^2(0)}-2} \tag{32}$$

where the ratio $\tilde{D}_0 \equiv \frac{2\mu^\eta(0)}{(\sigma^\eta)^2(0)}$ determines the existence of a degenerate distribution around $\eta \sim 0$. If $\tilde{D}_0$ goes below 2, then the stationary distribution features an infinite mass around $\eta_t = 0$ (potentially with other peaks).[21] If $\tilde{D}_0$ goes below 1, then the stationary distribution $d(\eta)$ becomes degenerate (i.e., Dirac-delta) around $\eta = 0$. Appendix B.2 offers an analytic proof, providing a threshold of $|\alpha^O - \alpha|$ that generates a permanent trap of net worth with zero recapitalization probability.[22] ■

Corollary 2 *The threshold level of belief that determines net worth trap in an economy without a short-selling constraint is given by*

$$\left|\alpha^O - \alpha\right| > \sigma\sqrt{\Gamma_0^2\sigma^2 + 2\Delta_0}, \tag{33}$$

While Proposition 3 provides an analytical characterization of the trap by studying the limit, Figure 1 displays the stationary distribution $d(\eta_t)$ throughout the state space.

Rational expectations equilibrium First, we observe that, under the benchmark rational expectations model (i.e., $\alpha^O = \alpha$), we obtain a degenerate stationary distribution with all the mass concentrated at $\eta_t = 1$. This is because experts, as more productive capital users, earn a high risk premium and take over the economy in the long run.[23]

[21] Brunnermeier (2024) relates the net worth trap to economic resilience, making a distinction between a trap with and without escape. When $\tilde{D}_0 < 2$, we have a trap with escape. The net worth trap when $\tilde{D}_0 < 1$ corresponds to the case without escape.

[22] $\mu^\eta(0) \equiv \lim_{\eta\to 0}\mu^\eta(\eta)$ and $\sigma^\eta(0) \equiv \lim_{\eta\to 0}\sigma^\eta(\eta)$ in (32) are from Lemma 1.

[23] Note that having the preference heterogeneity, including difference discount rates, might change this result. In particular, if $\rho^O > \rho^h$, the density is not degenerate. However, this is not crucial for our results and the main observation still holds - under the rational expectations equilibrium, there is no net worth trap for the experts and therefore there is no permanent inefficiency.

With belief distortions As experts become more optimistic, the mass in the crisis region becomes larger with sharper peaks around $\eta \simeq 0$. It is because when negative shocks shift the economy toward the inefficient region, a higher optimism level, on average, makes the economy more vulnerable to negative shocks through its effects on leverage, increasing its occupancy time in a crisis.

In fact, even small levels of optimism generates this effect in which all of the capital is allocated to inefficient agents (households) in the economy for most of the time. In a case where optimism is higher than a given threshold, in addition to this inefficient allocation, the probability of optimists to rebuild their net worth becomes zero, leading to the degenerate stationary distribution at $\eta_t = 0$. We call this phenomenon a "net worth trap", since the economy becomes trapped in this state and remains perennially inefficient. In summary, optimism shifts the original degenerate distribution at $\eta_t = 1$ to another degenerate distribution at $\eta_t = 0$. A similar effect occurs when experts become pessimistic, which we will discuss in the case of swinging beliefs in Section 2.5.1.

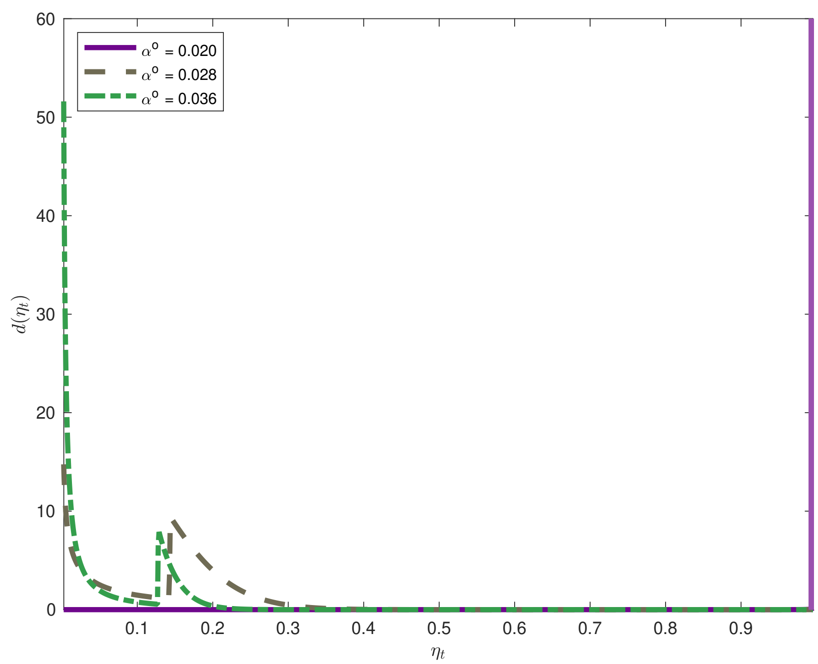
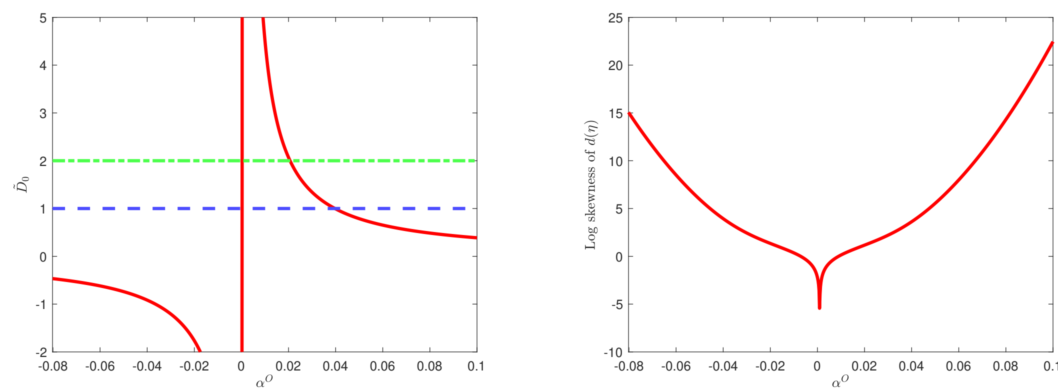


Figure 1: Stationary distribution of η_t and the net worth trap

Figure 2a plots $\tilde{D}_0$ against the expert's perceived long-run growth rate α^O when the true growth rate is $\alpha = 0.02$. With α^O close enough to true α, $\tilde{D}_0$ is above 2, indicating a non-degenerate stationary distribution with finite mass at the point $\eta \sim 0$. In this case, even when the economy enters a crisis, the relative growth rate of optimists' net worth compared with its volatility is high (i.e., expectation errors are small) enough to enable optimists to recapitalize. Under higher optimism, the quantity $\tilde{D}_0$ becomes smaller than the threshold value of 2, indicating that the density has an infinite mass at the point $\eta \sim 0$. Once optimism crosses a threshold value $\bar{\alpha}^O$, $\tilde{D}_0$ falls below 1, where the stationary distribution becomes a Dirac delta measure at $\eta = 0$. In this case, the relative growth rate of expert's wealth share compared with its volatility is negative (i.e., expectation errors are large) such that once the economy enters a crisis, the negative growth rate drains the wealth share until it reaches the point $\eta = 0$. From this point, the experts can no longer recapitalize, and the economy becomes trapped in this state inefficiently forever. In addition, we prove in Appendix B.3 that our result does not depend on the log-preference of agents, and is robust to stochastic differential utilities à la Duffie and Epstein (1992) under some conditions.[24]



(a) Tail analysis of stationary distribution

(b) Skewness of the distribution around $\eta \sim 0$

Figure 2: The blue dashed line in Figure 2a indicates the threshold value of $\tilde{D}_0 = 1$ below which the economy generates a net worth trap.

Corollary 3 *The asymptotic skewness of the stationary distribution $d(\eta)$ increases in optimism α^O.*

We can analyze the third moment of the distribution $d(\eta)$ when η approaches 0. Figure 2b draws the $\log$ skewness of the distribution around $\eta \sim 0$. The skewness increases with the experts' belief distortion $|\alpha^O - \alpha|$. The origin of the net worth trap lies in the endogenous risk premium and expectation errors of experts. There are two opposing forces in a crisis that affect the net worth of experts. First, the risk premium is higher in a crisis, which loads

[24] We assume risk aversion greater than one and the intertemporal elasticity of substitution (IES) equal to one in the recursive utility case for analytical tractability.

positively on the expected growth rate of the expert's net worth.[25] This is because experts are leveraged in risky capital and earn a premium for holding it, helping them recapitalize faster.

The other force is the expectation error from experts' beliefs about the long-run growth rate of productivity. When the belief distortion $|\alpha^O - \alpha|$ is large enough, the second expectation error channel dominates the first recapitalization channel. This effect drains the net worth of experts by increasing the endogenous risk and prohibiting the economy from escaping form the singularity point of zero expert net worth. While the magnitude of belief distortion around $\eta = 0$ is what matters for the net worth trap, it has an impact on the frequency of a crisis by affecting the dynamics of wealth share even at the steady state. This can be seen in Figure 3. Panel 3a shows that the drift $\mu^\eta(\eta)\eta$ becomes more negative in the efficient region when belief distortion increases. Moreover, its endogenous risk, captured by $\sigma^\eta(\eta)\eta$, increases in optimism during a crisis, but does not change with optimism in the efficient region, as shown in Figure 3b. The growth rate of expert's net wealth share (i.e., drift) in the efficient region relative to its endogenous volatility is a crucial variable determining the probability, or the frequency of a crisis.

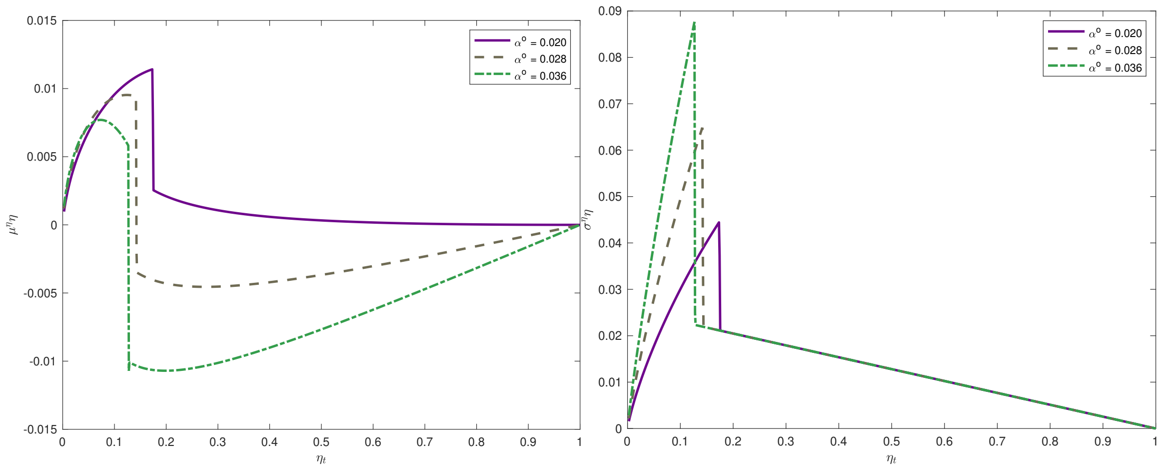
More generally, with larger belief distortion, the growth rate of the expert wealth share relative to its endogenous volatility becomes low enough (due to high expectation errors) to create a two-peaked stationary distribution with an infinite mass around $\eta = 0$. For higher optimism levels, the *relative* growth rate becomes low enough to force the probability of recapitalization of optimists to zero, formalized in Proposition 3.

Which beliefs matter? One interesting aspect of our framework is that even if experts are optimistic or pessimistic enough *only around* $\eta_t \sim 0$, a net worth trap arises.[26] For example, when the belief distortion is large around $\eta_t \sim 0$, once the economy shifts to around $\eta_t \sim 0$ due to a series of negative shocks, experts never recapitalize since the expectation error effect dominates the risk premium effect. We revisit this feature of our model later when we study the swinging beliefs in Section 2.5.2.

2.5.1 Special Cases

General preference In Appendix B.3, we relax the assumption of logarithmic preference specification and show that Proposition 2 is robust when agents have utility à la Duffie and

[25] Risk premium is high since capital price is low in crisis. This is shown in Section 2.6.1.
[26] We appreciate an anonymous referee for pointing out this feature of the model.



(a) $\mu_\eta(\eta_t) \cdot \eta_t$ as a function of η_t (b) $\sigma^\eta(\eta_t) \cdot \eta_t$ as a function of η_t

Figure 3: Wealth share dynamics: drift and volatility

Epstein (1992). The following proposition summarizes the result.

Proposition 4 *Assume that all agents have a Duffie-Epstein preference with a risk-aversion parameter γ. In this case, for intertemporal elasticity of substitution (IES) fixed at 1, there exists a threshold level of belief beyond which the economy features a net worth trap. For optimism, the threshold is given by*

$$\alpha^O - \alpha > -T_0 + \sqrt{T_0^2 - P_0}.$$

For pessimism, the threshold is given by

- *Without short-sale constraint:*

$$\alpha^O - \alpha < -T_0 - \sqrt{T_0^2 - P_0}. \tag{34}$$

- *With short-sale constraint:*

$$\alpha^O - \alpha < -\min\left\{T_0 + \sqrt{T_0^2 - P_0}, \max\left\{\sigma^2\left(\gamma + \Gamma_0\right), \Delta_0 + \frac{1}{2}\sigma^2\right\}\right\}, \tag{35}$$

where for Γ_0 and Δ_0 defined in Lemma 1,

$$T_0 = \left\{\Gamma_0 - \gamma\left(\gamma - 1 + \Gamma_0\right)\right\}\sigma^2,$$
$$P_0 = \sigma^2\left[\Gamma_0^2\sigma^2 - 2\gamma\left(\gamma - 1 + \Gamma_0\right)\Gamma_0\sigma^2 - 2\gamma^2\Delta_0\right].$$

Complete market If $l = 1$ and $\delta^H = \delta^O$, our model becomes a complete market model, as households are no longer in a disadvantageous position in production and investment efficiency. With homogeneous preferences, this also removes the desire to trade with each other. In this case, any amount of belief distortion leads to loss of entire net worth of experts in the long run. This result aligns with the "market selection hypothesis" à la Blume and Easley (2006) that agents with incorrect beliefs tend to lose wealth in the long run, and more recently, the result of Borovička (2020). We formalize this in the following proposition.

Proposition 5 *Under complete markets with $l = 1$ and $\delta^H = \delta^O$, if $\alpha^O \neq \alpha$, experts lose the entire wealth in the long run and the economy features a net worth trap.*

2.5.2 Swinging Beliefs

There is a large literature on how extrapolative beliefs drive asset returns, e.g., Bordalo et al. (2018). The idea of extrapolative expectations is that agents form beliefs based on recent shocks. In good times, agents become optimistic about the growth rate of economy, since the recent technological shocks have been positive. However, agents become pessimistic in bad times, since recent shocks have been negative. We now modify our dogmatic belief assumption to capture this effect. That is, we assume that in normal times, when capital is efficiently allocated to experts, optimism prevails. In bad times, when capital is misallocated to households, pessimism prevails. This simple modification allows us to capture the spirit of extrapolative expectations while still remaining tractable. The long-run growth rate perceived by optimists, denoted by O_t, has the following functional form:

$$O_t = \mathbf{1}_{\psi_t < 1} \cdot \alpha^P + \mathbf{1}_{\psi_t = 1} \cdot \alpha^O \tag{36}$$

where, $\alpha^P < \alpha < \alpha^O$. Note that O_t is higher than true α when the economy is in a stochastic steady state, but it becomes lower than α when the economy enters a crisis. Since the inefficient regime itself is determined endogenously, O_t captures a swinging sentiment in an endogenous manner, resonating with the diagnostic expectations literature (e.g., Bordalo et al. (2018) and Maxted (2023)): experts, more productive users of capital than households, are optimistic about growth in production efficiency in the stochastic steady state, thus raising leverage and pushing up capital prices. However, when the economy turns recessionary, their net worth drops, and their expectation of capital holding returns turns pessimistic. O_t following (36) captures the effects of diagnostic expectations in a simple manner. With this, we show that whether swinging beliefs stabilize or exacerbate the instability depends on how large the belief distortion is.

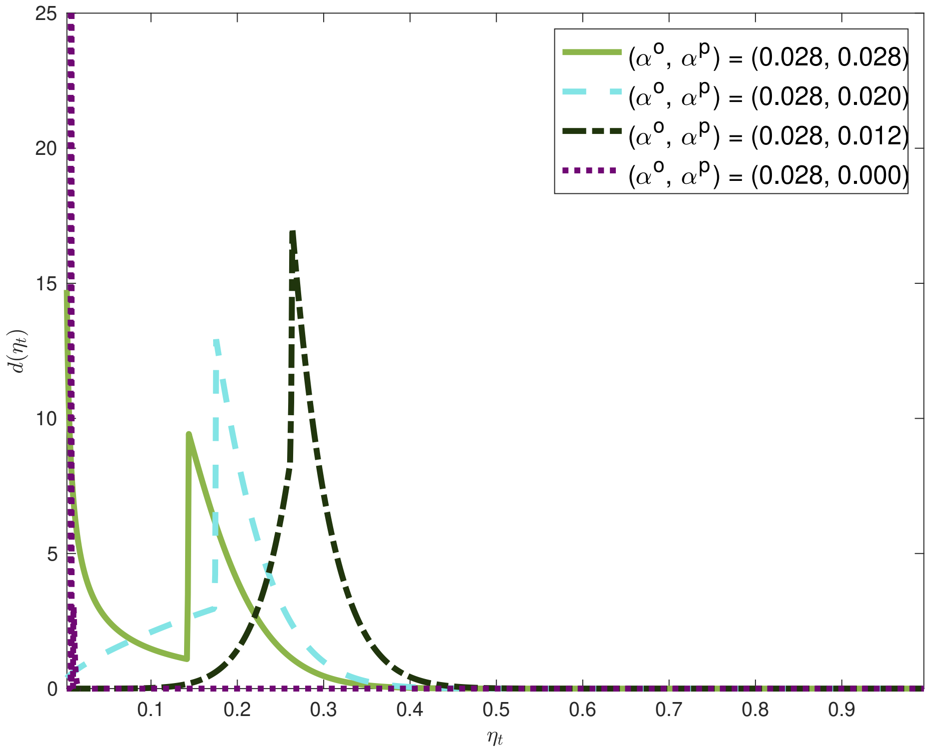


Figure 4: Stationary distribution of η_t under swinging beliefs

Stabilizing effect For low levels of pessimism, i.e., not too high $|\alpha^P - \alpha|$, the risk premium effect dominates the expectation error channel. That is, as the wealth share of experts is close to zero, the fact that risk premium is high loads positively on its net worth counteracting the loss in premium due to distorted leverage choice. We demonstrate this in Figure 4 by characterizing the stationary distribution of expert's wealth share. We assume that in (36) α^O is fixed at 2.8% while α^P takes $\{2\%, 1.2\%, 0\%\}$. For small belief distortions ($\alpha^P = 0.02$ and $\alpha^P = 0.012$), swinging beliefs shift the distribution to the right with less mass on the left tail. Another notable effect is that the economy stabilizes around the crisis threshold η^ψ for $0 < \alpha^p < \alpha^O$. This is because the economy enters a crisis state even when experts are well capitalized, lowering the intensity of fire sale and mitigating the amplification of risk in crisis periods. Lower amplification means that their expectation errors are smaller in a crisis, helping them to recapitalize faster.

Destabilizing effect However, if the degree of pessimism $|\alpha^P - \alpha|$ becomes large enough, it destabilizes the economy and the stationary distribution converges to a Dirac delta distribution around $\eta_t \sim 0$, that is, the economy has a net worth trap. As shown in Section 2.5, a large belief distortion around $\eta_t \sim 0$ prevents experts from earning a high risk premium, causing their net worth to be trapped at $\eta_t \sim 0$ as the rational agent earns the premium and takes over the economy. For $\alpha^P = 0$, we observe in Figure 4 that the stationary density is concentrated at $\eta = 0$. Thus, a swinging sentiment similar to diagnostic expectations might eliminate the possibility of a perpetual crisis or exacerbate it, depending on the degree of pessimistic beliefs in a crisis.

2.6 Other Model Behaviors

In this section, we explain the dynamics of the model with belief distortions. For expositional purposes, we focus on the optimism case, so here, experts are optimists.

2.6.1 Risk Amplification from the Optimism

Figure 5 presents the equilibrium quantities of the model for different levels of optimism.[27] In the economy's stochastic steady state, all capital is operated by optimists who are sufficiently capitalized with high η_t. Since the experts have a superior ability to manage capital, this efficient region of the economy features high levels of capital price with low levels of endogenous volatility. The q-theory result implies that the investment rate is high, and that in turn leads to higher growth rates in both capital and output.

A series of negative shocks then drains the net worth of optimists since they operate with leverage, as seen in Figure 5b. Once the wealth share falls sufficiently below a threshold, say η^ψ, optimists start fire-selling capital to the inefficient agents (i.e., rational households) who demand a high risk premium to hold the risky asset since they are not as productive in operating capital as experts. This inefficient crisis region features low levels of capital price, investment, and output, and high levels of endogenous volatility. These model aspects are common in many macro-finance models, e.g., Brunnermeier and Sannikov (2014, 2016b).

Optimism and the amplification of risks The role of optimism in our model is to change the equilibrium allocation in two ways: by affecting the threshold at which optimists fire-sell capital, and the amplification that the fire-selling generates. The effect of optimism is

[27] Figure A1 in Appendix A provides other equilibrium quantities not included in Figure 5.

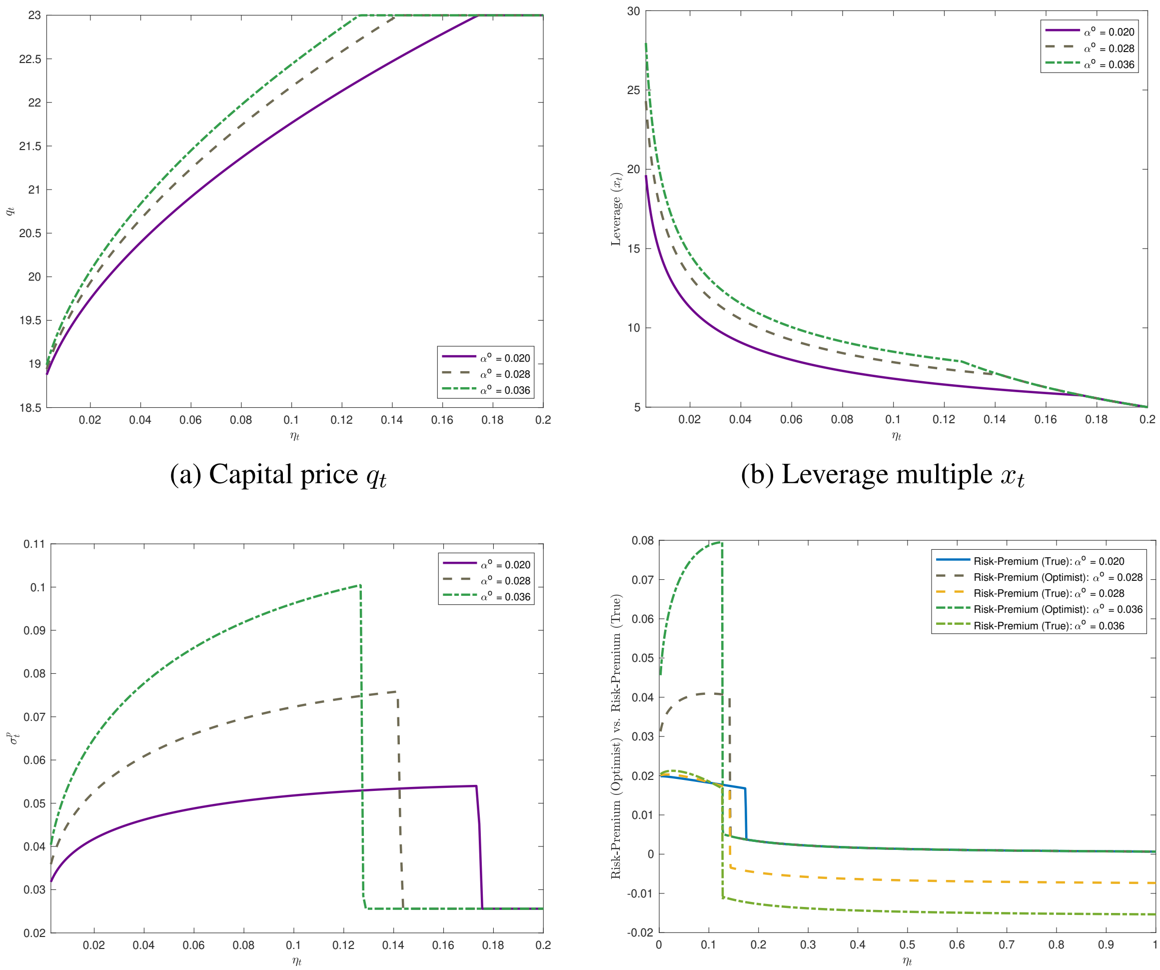
particularly salient in the crisis region with little effect around the stochastic steady state. For example, optimism $\alpha^O - \alpha$ does not affect the price of capital q_t in the steady state since optimists operate all capital and the 'risk-adjusted' return that the households obtain from holding capital is less than the prevailing risk-free rate.[28] Thus they prefer to invest in risk-free bonds issued by experts. However, in the inefficient region where households hold a portion of capital, the degree of optimism $\alpha^O - \alpha$ starts to matter. As optimism $\alpha^O - \alpha$ increases, the threshold for the efficient region η^ψ falls. That is, higher optimism leads to a larger part of the state space where optimists operate all capital by taking on more leverage. In particular, the fact that they take on more leverage in the normal region precipitates to a stronger fire-selling effect as evidenced by a steeper slope of capital price as seen in Figure 5a, characterizing the boom-bust cycles documented in the literature (see Krishnamurthy and Li (2024), for example).

The role of optimism as a risk amplifier on top of the financial accelerator role is seen by inspecting the endogenous capital price volatility σ_t^p shown in Figure 5c. As optimism increases, the endogenous volatility σ_t^p is more amplified in the crisis region. Actually, the endogenous risk can be written as

$$\sigma_t^p \left(1 - \left(x_t^O - 1 \right) \frac{\dfrac{dq(\eta_t)}{q(\eta_t)}}{\dfrac{d\eta_t}{\eta_t}} \right) \equiv \sigma_t^p \left(1 - \left(x_t^O - 1 \right) \varepsilon_{q,\eta} \right) = \sigma, \tag{37}$$

where $\varepsilon_{q,\eta}$ is defined as the elasticity of the price-earnings ratio (i.e., the normalized capital price) with respect to the wealth share of experts, η_t. We observe that optimism raises the capital price volatility through higher leverages (*leverage effect*). Second, greater optimism leads to a higher price elasticity (*elasticity effect*), captured by $\epsilon_{q,\eta}$, and magnifies the leverage effect. The elasticity effect arises due to the intense fire-selling of optimists explained earlier. In short, higher optimism causes a boom phase of excessive leverage, which leads to strong fire-sales of risky capital to the inefficient households during a crisis, contributing to a higher financial volatility σ_t^p in the economy's crisis region. One point worth noting is that we capture a belief-driven adverse loop: a higher optimism increases endogenous risk σ_t^p in a crisis, raising the degree of optimism in the expected capital return as seen in (17), which in turn amplifies the endogenous risk even further.

[28] We observe in Figure 5d that the *true* risk premium is negative around the stochastic steady state, which aligns with the fact that households do not hold capital at all in those regions. In contrast, the *perceived* risk premium of optimists is positive in both regions.



(a) Capital price q_t

(b) Leverage multiple x_t

(c) Endogenous volatility σ_t^p

(d) *Perceived* and *true* risk-premium

Figure 5: Equilibrium variables as functions of η_t

Lastly, when optimists are sufficiently capitalized, greater optimism leads to a higher leverage ratio of experts and a higher interest rate, as seen in Figure A1c of Appendix A. In the crisis region where households hold some capital, higher optimism creates a stronger precautionary saving motive due to a greater endogenous volatility, pushing down the risk free rate.[29]

These effects explained so far stem from that the risk premium perceived by optimists are different from the true level of risk premium. This expectation error affects the net worth

[29]When optimists start deleveraging and fire-sell their capital assets, the interest rate r_t drops in a discontinuous manner. It is anticipated: as optimists suddenly fire-sell the capital and reduce their leverage demand, the risk-free rate should drop to clear the bond market. After the economy enters the inefficient region, i.e., $\eta_t \leq \eta^{\psi}$, the interest rate level becomes continuous again.

of optimists strong enough to generate an amplified boom-bust cycle. As seen in Figure 5d, optimism makes the more productive experts to take on excessive leverage precisely because they perceive the risk premium level to be higher than it actually is.

The expectation error stemming from optimism can drain the net worth of optimists at the stochastic steady state, and raise the occupation time of economy in a crisis. In fact, the static comparative plot in Figure 5 reveals that there is a smaller measure of the state space where capital is misallocated to unproductive households from fire-sales, when optimism is higher. However, since the expectation errors, by which optimists' perceived risk-premium is higher than its true level, are increasing in the optimism degree, their net worth can be drained at a high rate, trapping their wealth share at low levels and causing inefficiency for a long time, as studied in Section 2.5.

In Appendix B.4, we study the welfare implications of optimism for rational households and find that optimism can lower the welfare level of households through multiple channels, including the capital misallocation channel.

3 Conclusion

In this paper, we build a continuous-time economy with experts and households, where agents have heterogeneous beleifs on the long-run growth rate of the economy. Experts, who are more productive in managing capital, hold dogmatic beliefs and agree to disagree with households who are less productive. We characterize a phenomenon called the 'net worth trap' that arises when experts hold different beliefs about the economic growth rate compared to the true growth rate. The trap occurs because large belief distortions of experts increase the endogenous risk to the point that the economy cannot escape from the zero-net-worth point in the state space. The mechanisms are driven by two opposing forces: (i) risk premium channel that loads positively on the expected growth rate of expert's wealth, and (ii) expectation error channel that loads negatively on their expected growth rate of wealth. For small belief distortions, the first channel dominates helping the experts recapitalize eventually. For large belief distortions, the second effect dominates.

A swing in beliefs, as in the case of extrapolative expectations, eliminates or exacerbates the net worth trap, depending on the level of belief distortion in a crisis. Therefore, how productive agents form beliefs has important consequences for financial stability and asset price behaviors in the long run.

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Declaration of generative AI in scientific writing

During the preparation of this work, the authors did not use generative AI or AI-assisted technologies.

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References

Basak, Suleyman, "A Model of Dynamic Equilibrium Asset Pricing with Heterogeneous Beliefs and Extraneous Risk," *Journal of Economic Dynamics and Control*, 2000, *24* (1), 63–95.

_ **and Benjamin Croitoru**, "Equilibrium Mispricing in a Capital Market with Portfolio Constraints," *The Review of Financial Studies*, 2000, *13* (3), 715–748.

_ **and** _ , "On the Role of Arbitrageurs in Rational Markets," *Journal of Financial Economics*, 2006, *81* (1), 143–173.

Beutel, Johannes and Michael Weber, "Beliefs and Portfolios: Causal Evidence," Technical Report, Working paper 2022.

Blume, Lawrence and David Easley, "If You're so Smart, Why Aren't You Rich? Belief Selection in Complete and Incomplete Markets," *Econometrica*, 2006, *74* (4), 929–966.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer, "Diagnostic Expectations and Credit Cycles," *Journal of Finance*, 2018, *73* (1), 199–227.

_ **,** _ **, Rafael La Porta, Matthew O'Brien, and Andrei Shleifer**, "Long Term Expectations and Aggregate Fluctuations," *NBER Working Paper*, August 2023.

Borovička, Jaroslav, "Survival and Long-Run Dynamics with Heterogeneous Beliefs under Recursive Preferences," *Journal of Political Economy*, 2020, *128* (1), 206–251.

Brunnermeier, Markus K., "Macrofinance and Resilience," *Journal of Finance*, 12/2024 2024, *79.*

_ **and Y. Sannikov**, "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 2014, *104* (2), 379–421.

_ **and Yuliy Sannikov**, "Macro, Money, and Finance: A Continuous Time Approach," *Handbook of Macroeconomics*, 2016b, ***2***, 1497–1545.

Caballero, Ricardo J and Alp Simsek, "A Risk-centric Model of Demand Recessions and Speculation," *The Quarterly Journal of Economics*, 2020, *135* (3), 1493–1566.

Camous, Antoine and Alejandro Van der Ghote, "Financial Stability and Financial Regulation under Diagnostic Expectations," *Working Paper*, 2023.

Candia, Bernardo, Olivier Coibion, and Yuriy Gorodnichenko, "The Macroeconomic Expectations of Firms," *Handbook of Economic Expectations*, 2021.

Chabakauri, Georgy, "Asset pricing with heterogeneous preferences, beliefs, and portfolio constraints," *Journal of Monetary Economics*, 2015, *75*, 21–34.

Coibion, Olivier, Yuriy Gorodnichenko, and Tiziano Ropele, "Inflation Expectations and Firm Decisions: New Causal Evidence," *Quarterly Journal of Economics*, 2020, *135*, 165–219.

Cooper, Ilan and Richard Priestley, "Time-Varying Risk Premiums and the Output Gap," *The Review of Financial Studies*, 2009, *22* (7), 2801–2833.

Coval, Joshua D. and Anjan V. Thakor, "Financial intermediation as a beliefs-bridge between optimists and pessimists," *Journal of Financial Economics*, 2005, *75* (3), 535–569.

Detemple, Jerome and Shashidhar Murthy, "Equilibrium Asset Prices and no-Arbitrage with Portfolio Constraints," *The Review of Financial Studies*, 1997, *10* (4), 1133–1174.

DeVault, Luke, Richard Sias, and Laura Starks, "Sentiment metrics and investor demand," *The Journal of Finance*, 2019, *74* (2), 985–1024.

Dong, Ding, Zheng Liu, and Pangfei Wang, "A Macroeconomic Model with Speculation," Technical Report, Working paper 2022.

Duffie, Darrell, "Presidential Address: Asset Price Dynamics with Slow-Moving Capital," *Journal of Monetary Economics*, 2010, *65* (8), 1237–1267.

_ **and Larry G. Epstein**, "Stochastic Differential Utility," *Econometrica: Journal of the Econometric Society*, 1992, *60* (2), 353–394.

Gallmeyer, Michael and Burton Hollifield, "An Examination of Heterogeneous Beliefs with a Short-Sale Constraint in a Dynamic Economy," *Review of Finance*, 2008, *12* (2), 323–364.

Geanakoplos, John, "The Leverage Cycle," *NBER macroeconomics annual*, 2010, *24* (1), 1–66.

Gertler, Mark, Nobuhiro Kiyotaki, and Andrea Prestipino, "Credit booms, financial crises, and macroprudential policy," *Review of Economic Dynamics*, 2020, *37* (1), 8–33.

Giglio, Stefano, Matteo Maggiori, Johannes Stroebel, and Stephen Utkus, "Five Facts about Beliefs and Portfolios," Technical Report, National Bureau of Economic Research 2019.

Gopalakrishna, Goutham, "A Macro-Finance Model with Realistic Crisis Dynamics," Technical Report, Working paper 2022.

Harrison, J Michael and David M Kreps, "Speculative investor behavior in a stock market with heterogeneous expectations," *The Quarterly Journal of Economics*, 1978, *92* (2), 323–336.

He, Zhiguo and Arvind Krishnamurthy, "Intermediary Asset Pricing," *American Economic Review*, 2013, *103* (2), 732–70.

_, Bryan Kelly, and Asaf Manela, "Intermediary asset pricing: New evidence from many asset classes," *Journal of Financial Economics*, 2017, *126* (1), 1–35.

Krishnamurthy, Arvind and Professor Wenhao Li, "Dissecting Mechanisms of Financial Crises: Intermediation and Sentiment," *Journal of Political Economy*, 2024.

Maxted, Peter, "A Macro-Finance Model with Sentiment," *The Review of Economic Studies*, 2023.

Merton, Robert C., "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, 1971, *3*, 373–413.

Puri, Manju and David T. Robinson, "Optimism and economic choice," *Journal of Financial Economics*, 2007, *86* (1), 71–99.

Rapach, David E., Jack K. Strauss, and Guofu Zhou, "Out-of-Sample Equity Premium Prediction: Combination Forecasts and Links to the Real Economy," *The Review of Financial Studies*, 2012, *23* (2), 821–862.

Romer, Christina D., "The Nation in Depression," *Journal of Economic Perspectives*, 1993, *7* (2), 19–39.

Scheinkman, Jose A and Wei Xiong, "Overconfidence and speculative bubbles," *Journal of political Economy*, 2003, *111* (6), 1183–1220.

Schmitt-Grohé, Stephanie and Martín Uribe, "Optimal simple and implementable monetary and fiscal rules," *Journal of Monetary Economics*, September 2007, *54* (6), 1702–1725.

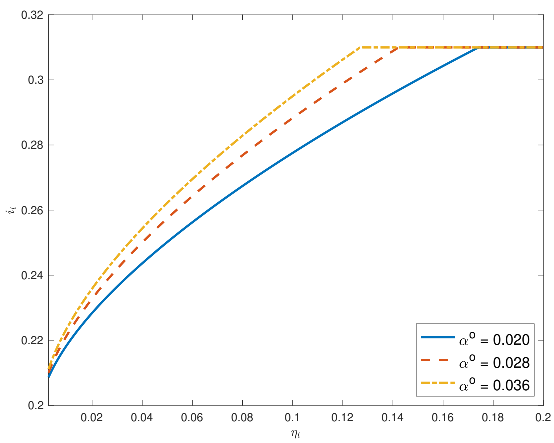
Simsek, Alp, "Belief Disagreements and Collateral Constraints," *Econometrica*, 2013, *81* (1), 1–53.

Weber, Michael, Francesco D'Acunto, Yuriy Gorodnichenko, and Olivier Coibion, "The Subjective Inflation Expectations of Households and Firms: Measurement, Determinants, and Implications," *Journal of Economic Perspectives*, 2022, *36.*

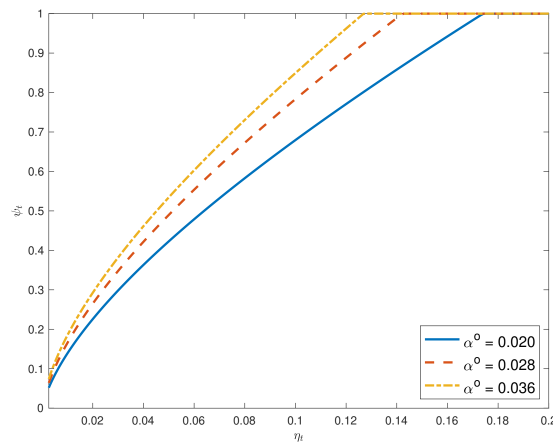
Welch, Ivo, "Views of financial economists on the equity premium and on professional controversies," *The Journal of Business*, 2000, *73* (4), 501–537.

Yan, Hongjun, "Natural Selection in Financial Markets: Does it work?," *Management Science*, 2008, *54* (11), 1935–1950.

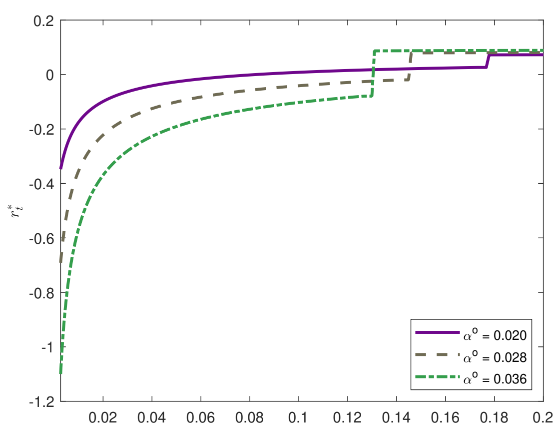
Appendix A Additional Figures



(a) Investment rate ι_t as a function of η_t



(b) Capital share ψ_t



(c) Equilibrium interest rate r_t^*

Figure A1: Other equilibrium variables as functions of η_t

Appendix B Proofs and Derivations

Proof of Proposition 1. We separately solve problems of experts and households.

Experts Since experts believe that dZ_t^O, not dZ_t, is the true Brownian motion driving the business cycle, they believe the wealth share η_t follows:

$$\frac{d\eta_t}{\eta_t} = \left(\mu_t^\eta(\eta_t) + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^\eta(\eta_t)\right)dt + \sigma_t^\eta(\eta_t)dZ_t^O, \tag{B.1}$$

which is consistent with the true η_t process in (27). Based on Merton (1971), we conjecture her value function $V(\cdot)$ will depend on her own wealth w_t^O and the aggregate wealth share of experts η_t with the following form:

$$V\left(w_t^O, \eta_t\right) = \frac{\log w_t^O}{\rho} + f\left(\eta_t\right). \tag{B.2}$$

Based on (18) and (B.1), the Hamiltonian-Jacobi-Bellman (HJB) equation for an individual expert's problem can be written as[1]

$$\rho V(\cdot) = \max_{x_t^O, c_t^O, i_t^O} \log c_t^O + \left[w_t^O\left(x_t^O\left(\frac{\mathbb{E}_t^O\left(dr^{Ok}\right)}{dt}\right) + (1 - x_t^O)r_t\right) - c_t^O\right]\frac{dV_t}{dw_t^O}$$
$$+ \frac{\left(x_t^O w_t^O \sigma_t^p\right)^2}{2}\frac{d^2V_t}{d\left(w_t^O\right)^2} + \left(\eta_t\left(\mu_t^\eta(\eta_t) + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^\eta(\eta_t)\right)\right)\frac{dV_t}{d\eta_t} + \frac{\left(\eta_t\sigma_t^\eta(\eta_t)\right)^2}{2}\frac{d^2V_t}{d\eta_t^2}.$$

The first order condition[2] with respect to c_t^O are given by:

$$\frac{1}{c_t^{O*}} = \frac{dV_t}{dw_t^O}\left(w_t^O, \eta_t\right) = \frac{1}{\rho w_t^O}, \tag{B.3}$$

which gives $c_t^{O*} = \rho w_t^O$ at optimum. As every aggregate variable including q_t, i_t^O, μ_t^p, σ_t^p, x_t^O, and r_t will be expressed as functions of the aggregate wealth share η_t, Merton (1971)

[1] Due to the assumed value function in (B.2), we ignore the cross-derivative term of $V(\cdot)$ with respect to w_t^O and η_t.

[2] We use the following relation from equation (17):

$$\mathbb{E}_t^O\left(dr^{Ok}\right) = \left(\frac{\gamma_t^O - \iota_t^O\gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p\right)dt = \left(\frac{1 - \iota_t^O}{q_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p\right)dt.$$

justifies our choice of value function in (B.2). The first-order condition with respect to x_t^O gives the optimal portfolio choice given in Proposition 1.

To derive i_t^{O*}, we observe that the investment appears only in a intra-temporal manner. Thus the corresponding optimization problem is static and satisfies the first-order condition: $\Lambda^{O\prime}(i_t) = q_t^{-1}$ (i.e., marginal Tobin's q). With our $\Lambda^O(i_t)$, we finally obtain:

$$i_t^{O*}(q_t) = \frac{q_t^2 - 1}{2k}. \tag{B.4}$$

Rational households Since households have the rational expectations, i.e., they believe that dZ_t, not dZ_t^O, is the true Brownian motion driving the business cycle, they believe the wealth share η_t follows:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta(\eta_t)dt + \sigma_t^\eta(\eta_t)dZ_t, \tag{B.5}$$

which is consistent with the true η_t process in (27). Based on Merton (1971), we conjecture her value function $\underline{V}(\cdot)$ will depend on her own wealth w_t^H and the aggregate wealth share of experts η_t as the state variable:

$$V^H\left(w_t^H, \eta_t\right) = \frac{\log w_t^H}{\rho} + f^H\left(\eta_t\right). \tag{B.6}$$

Based on (20) and (B.5), the Hamiltonian-Jacobi-Bellman (HJB) equation for an individual expert's problem can be written as[3]

$$\begin{aligned}\rho V^H(\cdot) = \max_{x_t^H \geq 0, c_t^H, i_t^H} \log c_t^H &+ \left[w_t^H\left(x_t^H\left(\frac{\mathbb{E}_t^H\left(dr^{Hk}\right)}{dt}\right) + (1 - x_t^H)r_t\right) - c_t^H\right]\frac{dV_t^H}{dw_t^H} \\ &+ \frac{\left(x_t^H w_t^H \sigma_t^p\right)^2}{2}\frac{d^2V_t^H}{d\left(w_t^H\right)^2} + \eta_t\left(\mu_t^\eta(\eta_t)\right)\frac{dV_t^H}{d\eta_t} + \frac{\left(\eta_t\sigma_t^\eta(\eta_t)\right)^2}{2}\frac{d^2V_t^H}{d\eta_t^2}.\end{aligned}$$

The first order condition with respect to c_t^G are given by:

$$\frac{1}{c_t^{H*}} = \frac{dV_t^H}{dw_t^H}\left(w_t^H, \eta_t\right) = \frac{1}{\rho w_t^H}, \tag{B.7}$$

[3]We use the following relation:

$$\mathbb{E}_t^H\left(dr^{Hk}\right) = \mathbb{E}_t\left(dr^{Hk}\right) = \left(\frac{\gamma_t^H - \iota_t^H\gamma_t^H}{p_t} + \Lambda^H(\iota_t^H) - \delta^H + \mu_t^p\right)dt.$$

which gives $c_t^{H*} = \rho w_t^H$ at optimum. As aggregate variables including q_t, i_t^H, μ_t^p, σ_t^p, x_t^H, and r_t are expressed as functions of the aggregate wealth share η_t, Merton (1971) justifies our choice of value function in (B.6). The first-order condition with respect to x_t^H gives the optimal portfolio choice given in Proposition 1.

To derive ι_t^H, we observe that the investment appears only in a intra-temporal manner. Thus the corresponding optimization problem is static and satisfies the first-order condition: $\Lambda^{H'}(i_t) = l \cdot q_t^{-1}$. With $\Lambda^H(\cdot) = l \cdot \Lambda^H(\cdot)$, we finally obtain:

$$i_t^{H*}(q_t) = \frac{q_t^2 - 1}{2k}, \tag{B.8}$$

which is the same as $i_t^{O*}(q_t)$ in (B.4).

■

Proof of Proposition 2. The aggregate wealth of experts, W_t^O evolves with the process

$$dW_t^O = r_t W_t^O dt + \psi_t p_t K_t \left(dr_t^{Ok} - r_t dt\right) - c_t^O dt, \tag{B.9}$$

where $c_t^O = \rho W_t^O$ holds at optimum. By applying Ito's quotient rule on (25),[4] we have

$$\frac{d\eta_t}{\eta_t} = \frac{dW_t^O}{W_t^O} - \frac{d\left(p_t K_t\right)}{p_t K_t} + \left(\frac{d\left(p_t K_t\right)}{p_t K_t}\right)^2 - \frac{dW_t^O}{W_t^O}\frac{d\left(p_t K_t\right)}{p_t K_t}. \tag{B.10}$$

In addition, from the process (24) of the aggregate capital K_t, we obtain

$$\begin{aligned} \frac{1}{K_t}\frac{dK_t}{dt} &= \left(\Lambda^O\left(i_t^O\right) - \delta^O\right)\psi_t + \left(\Lambda^H\left(i_t^H\right) - \delta^H\right)(1-\psi_t) \\ &= \left(\Lambda^O\left(i_t^O\right) - \delta^O\right) - \left(\delta^H - \delta^O\right)(1-\psi_t) - (1-\psi_t)(1-l)\Lambda^O\left(i_t^O\right), \end{aligned} \tag{B.11}$$

where we used the property that $i_t^O = i_t^H$ in equilibrium as seen in (B.4) and (B.8). Applying Ito's product rule to the price process (8) and the capital process (B.11), and comparing

[4] Ito's quotient rule states that:

$$\frac{d\left(XY^{-1}\right)}{XY^{-1}} = \frac{dX}{X} - \frac{dY}{Y} + \left(\frac{dY}{Y}\right)^2 - \frac{dX}{X}\frac{dY}{Y}.$$

with (17), we obtain

$$\frac{d\left(p_t K_t\right)}{p_t K_t} = dr_t^{Ok} - \frac{1-\iota_t^O}{q_t}dt - \left(\delta^H - \delta^O\right)\left(1-\psi_t\right)dt - (1-\psi_t)(1-l)\Lambda^O\left(i_t^O\right)dt. \quad \text{(B.12)}$$

Since $\mathbb{E}_t^O\left(dr_t^{Ok}\right) - r_t = x_t^O\left(\sigma_t^p\right)^2$ from the experts' optimal portfolio decision à la Merton (1971), and from the fact that

$$\mathbb{E}_t^O\left(dr_t^{Ok}\right) = \mathbb{E}_t\left(dr_t^{Ok}\right) + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^p dt, \quad \text{(B.13)}$$

where $\mathbb{E}_t$ is the operator corresponding to the rational expectations, we plug (B.9), (B.12), and (B.13) into (B.10) and obtain

$$\begin{aligned}\frac{d\eta_t}{\eta_t} &= \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 dt - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p dt + \frac{1-\iota_t^O}{q_t}dt + \left(1-\psi_t\right)\left(\delta^H - \delta^O\right)dt \\ &\quad + \left(1-l\right)\left(1-\psi_t\right)\Lambda^O\left(i_t^O\right)dt - \rho dt + \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p dZ_t,\end{aligned} \quad \text{(B.14)}$$

where we used $c_t^O = \rho W_t^O$ in the equilibrium. From (B.14), we obtain

$$\mu^\eta(\eta_t) = \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p + \frac{1-\iota_t^O}{q_t} + (1-\psi_t)\left(\delta^H - \delta^O\right) + (1-l)\left(1-\psi_t\right)\Lambda^O(i_t^O) - \rho, \quad \text{(B.15)}$$

and

$$\sigma^\eta(\eta_t) = \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p. \quad \text{(B.16)}$$

■

Proposition 6 *When q_t is a function of state variable η_t with $q_t = q(\eta_t)$, the drift μ_t^p of the price of capital p_t is given by*

$$\mu_t^p = \alpha + \frac{q'\left(\eta_t\right)}{q_t}\mu^\eta(\eta_t)\eta_t + \frac{1}{2}\sigma^\eta(\eta_t)\eta_t\frac{q''(\eta_t)}{q(\eta_t)} + \sigma\sigma^\eta(\eta_t)\eta_t\frac{q'\left(\eta_t\right)}{q(\eta_t)}, \quad \text{(B.17)}$$

where $\sigma_t^\eta\left(\eta_t\right)$ is given by Proposition 2, and the volatility of the price of capital, σ_t^p, is given by

$$\sigma_t^p = \frac{\sigma}{1 - \left(x_t\eta_t - \eta_t\right)\dfrac{q'\left(\eta_t\right)}{q(\eta_t)}}. \quad \text{(B.18)}$$

Proof of Proposition 6. Applying Ito's lemma to the Markov relationship $q_t = q\left(\eta_t\right)$, we obtain

$$dq_t = q'(\eta_t)\, d\eta_t + \frac{1}{2} q''(\eta_t)\,(d\eta_t)^2 . \tag{B.19}$$

Plugging in equation (27) to (B.19) and using $(d\eta_t)^2 = \eta_t^2 \sigma^\eta(\eta_t)^2 dt$, we have

$$\frac{dq_t}{q_t} = \left(\eta_t \mu^\eta(\eta_t) \frac{q'(\eta_t)}{q(\eta_t)} + \frac{\eta_t^2 \sigma^\eta(\eta_t)^2}{2} \frac{q''(\eta_t)}{q(\eta_t)} \right) dt + \eta_t \sigma^\eta(\eta_t) \frac{q'(\eta_t)}{q(\eta_t)} dZ_t . \tag{B.20}$$

With the definition of normalized asset price (i.e., price-earnings ratio) $q_t = \frac{p_t}{\gamma_t^O}$, we have:[5]

$$\frac{dq_t}{q_t} = \left(\mu_t^p - \alpha + \sigma^2 - \sigma_t^p \sigma \right) dt + \left(\sigma_t^p - \sigma \right) dZ_t . \tag{B.21}$$

Comparing (B.20) and (B.21) yields:

$$\sigma_t^p = \sigma + \eta_t \sigma^\eta(\eta_t) \frac{q'(\eta_t)}{q(\eta_t)}, \tag{B.22}$$

and

$$\mu_t^p = \alpha + \eta_t \mu^\eta(\eta_t) \frac{q'(\eta_t)}{q(\eta_t)} + \frac{\eta_t^2 \sigma^\eta(\eta_t)^2}{2} \frac{q''(\eta_t)}{q(\eta_t)} + \sigma \sigma^\eta(\eta_t) \eta_t \frac{q'(\eta_t)}{q(\eta_t)} . \tag{B.23}$$

From (B.16) in Proposition 2, we have that

$$\sigma^\eta(\eta_t) = \frac{x_t \eta_t - \eta_t}{\eta_t} \sigma_t^p, \tag{B.24}$$

where we used $x_t \eta_t = \psi_t$. Therefore, by (B.22) and (B.24) we have

$$\sigma_t^p = \frac{\sigma}{1 - (x_t \eta_t - \eta_t) \dfrac{q'(\eta_t)}{q(\eta_t)}} . \tag{B.25}$$

■

B.1 Price of Capital and the Optimal Leverages

We derive a closed formed solution for leverage multiple and a first-order differential equation for the price of capital. The following summarize the main results of the paper.

[5]Note that in equation (B.21), we use the 'true' process for γ_t^O given in (12).

Proposition 7 *The equilibrium domain consists of two intervals $[0, \eta^\psi)$, where $\psi(\eta) < 1$, and $[\eta^\psi, 1]$ where $\psi(\eta) = 1$. The capital price $q(\eta)$ in our equilibrium is given by:*

$$q(0) = l \cdot \frac{1 - \iota(q(0))}{\rho} \tag{B.26}$$

and

$$q(\eta) = \frac{1 - \iota(q(\eta))}{\rho}, \ \ \forall \eta \in [\eta^\psi, 1] \tag{B.27}$$

The following procedures can be used to compute $\psi(\eta)$ and $q'(\eta)$ from $(\eta, q(\eta))$ to $(\eta^\psi, q(\eta^\psi))$ when $\eta < \eta^\psi$:

1. *Find ψ that satisfies*

$$\rho q(\eta) = \psi + (1-\psi)l - \iota(q)\left(\psi + (1-\psi)l\right) \tag{B.28}$$

2. *Compute $q(\eta)$ where $q(\eta)$ is given by the solution of the equation:*

$$\frac{(1-l)(1-\iota(q(\eta)))}{q(\eta)} + (1-l)\Lambda^O\left(\iota(q(\eta))\right) + \delta^H - \delta^O + \frac{\alpha^O - \alpha}{\sigma}\sigma^p(\eta) = \left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2, \tag{B.29}$$

where $\sigma^p(\eta)$ is given by (B.25). From (B.29), $\sigma^p(\eta)$ can also be expressed as:

$$\sigma^p(\eta) = \frac{\frac{\alpha^O-\alpha}{\sigma} + \sqrt{\left(\frac{\alpha^O-\alpha}{\sigma}\right)^2 + 4\left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right)\left(\frac{(1-l)(1-\iota(q))}{q(\eta)} + (1-l)\Lambda^O(\iota(q)) + \delta^H - \delta^O\right)}}{2\left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right)}, \tag{B.30}$$

where $q = q(\eta)$.

Proof of Proposition 7. From the good market equilibrium condition (23), we have

$$k_t^O\left(\gamma_t^O - \iota_t^O\gamma_t^O\right) + k_t^H\left(\gamma^H - \iota_t^H\gamma^H\right) = c_t^O + c_t^H, \ \ \forall t \in [0, \infty). \tag{B.31}$$

Observing that $c_t^O = \rho w_t^O$ and $c_t^H = \rho w_t^H$ in equilibrium, we divide (B.31) by $p_t K_t$ and obtain[6]

$$\rho q = \psi + (1-\psi)l - \iota(q)\left(\psi + (1-\psi)l\right). \tag{B.32}$$

[6] Since $\iota_t^H(q_t) = \iota_t^O(q_t)$, we use the notation $\iota(q(\eta))$ to denote that $\iota(\cdot)$ is a function of normalized asset price q, which in turn depends on the state variable η.

Since $\mathbb{E}_t^O\left(dr_t^{Ok}\right) - r_t = x_t^O\left(\sigma_t^p\right)^2$ from the experts' optimal portfolio decision, we obtain

$$\frac{1-\iota(q(\eta))}{q(\eta)} + \Lambda^O(\iota(q(\eta))) - \delta^O + \mu_t^p + \frac{\alpha^O - \alpha}{\sigma}\sigma^p(\eta) - r_t = \left(\frac{\psi}{\eta}\right)\sigma^p(\eta)^2. \tag{B.33}$$

For households, from Proposition 1, it must be the case where

$$l \cdot \frac{1-\iota(q(\eta))}{q(\eta)} + l \cdot \Lambda^O(\iota(q(\eta))) - \delta^H + \mu_t^p - r_t \leq \left(\frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2. \tag{B.34}$$

with equality when $\psi < 1$. Finally, by subtracting (B.34) from (B.33), we get

$$\frac{(1-l)(1-\iota(q(\eta)))}{q(\eta)} + (1-l)\Lambda^O\left(\iota(q(\eta))\right) + \delta^H - \delta^O + \frac{\alpha^O - \alpha}{\sigma}\sigma^p(\eta) = \left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2, \tag{B.35}$$

when $\psi_t < 1$, i.e., households hold some physical capital.

■

B.2 Stationary Distribution

We now compute the stationary (i.e., ergodic) distribution of the wealth share of experts, η_t. Given the η_t process in equation (27), the Kolmogorov forward equation (KFE) is given by

$$0 = -\frac{\partial}{\partial\eta}\left(\mu^\eta(\eta)\eta \cdot d(\eta)\right) + \frac{1}{2}\frac{\partial^2}{\partial\eta^2}\left(\left(\sigma^\eta(\eta)\eta\right)^2 d(\eta)\right). \tag{B.36}$$

where $d(\eta)$ denotes the distribution. With transformation $D(\eta) \equiv \left(\sigma^\eta(\eta)\eta\right)^2 \cdot d(\eta)$, equation (B.36) can be written as

$$\frac{D'(\eta)}{D(\eta)} = 2\frac{\mu^\eta(\eta)\eta}{\left(\sigma^\eta(\eta)\eta\right)^2}, \tag{B.37}$$

which can be solved easily by integrating both sides of (B.37).

Deriving $\mu^\eta(0)$ and $\sigma^\eta(0)$ in (32) and Proof of Lemma 1 The asymptotic solution of drift $\mu^\eta(\eta)$ and volatility $\sigma^\eta(\eta)$ of η, when $\eta \to 0$ is computed in a similar way to Brunnermeier and Sannikov (2014). Asymptotically, let us assume that as $\eta \to 0$, $q(\eta) \to q_0$ where q_0 is the equilibrium level of normalized capital price when households hold the entire aggregate wealth. Let as assume $\psi(\eta) \simeq \psi_0\eta$ when $\eta \sim 0$. Then we know $\sigma_t^p \to \sigma$ (from (B.18)), $\mu_t^p \to \alpha$ (from (B.17)), and $\sigma^\eta \to (\psi_0 - 1)\sigma$ thereafter. Following steps are needed to calculate $\mu^\eta(0)$ and $\sigma^\eta(0)$:

Step 1 From (B.26), i.e., $\rho q_0 = l(1-\iota_o)$ we obtain q_0 and $\iota_0 = \iota(q_0) = \frac{q_0^2-1}{2k}$.

Step 2 From (27), we know that as $\eta \to 0$, $\mu^\eta(\eta) \to \mu^\eta(0)$, where $\mu^\eta(0)$ can be written as

$$\mu^\eta(0) = ((\psi_0-1)\sigma)^2 - (\psi_0-1)\frac{\alpha^O-\alpha}{\cancel{\sigma}}\cancel{\sigma} + \frac{1-\iota_0}{q_0} + (\delta^H-\delta^O) + (1-l)\Lambda^O(\iota_0) - \rho.$$

Step 3 From (B.29), we know

$$\frac{(1-l)(1-\iota_0)}{q_0} + (1-l)\Lambda^O(\iota_0) + (\delta^H-\delta^O) + \frac{\alpha^O-\alpha}{\cancel{\sigma}}\cancel{\sigma} = (\psi_0-1)\sigma^2 \tag{B.38}$$

as

$$\left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right) \to \psi_0 - 1.$$

Step 4 Therefore from the above 3 steps, we calculate μ_0 and ψ_0, from which we calculate $\sigma^\eta(0^+) = (\psi_0-1)\sigma$.

$$\mu^\eta(0) \equiv \lim_{\eta\to 0} \mu^\eta(\eta)$$
$$\sigma^\eta(0) \equiv \lim_{\eta\to 0} \sigma^\eta(\eta) = (\psi_0-1)\sigma$$

Proof of Proposition 3. We know from the above Step 1 that q_0, ι_0, $\Lambda^O(\iota_0)$ are determined. By defining

$$\Gamma_0 \equiv \frac{1}{\sigma^2}\left[(1-l)\frac{1-\iota_0}{q_0} + (\delta^H-\delta^O) + (1-l)\Lambda^O(\iota_0)\right], \tag{B.39}$$

we obtain from Step 3

$$\psi_0 - 1 = \frac{\alpha^O-\alpha}{\sigma^2} + \Gamma_0. \tag{B.40}$$

From Step 2 and defining

$$\Delta_0 \equiv \frac{1-\iota_0}{q_0} + (\delta^H-\delta^O) + (1-l)\Lambda^O(\iota_0) - \rho, \tag{B.41}$$

it follows that

$$\begin{aligned}\mu^{\eta}(0^{+}) &= (\psi_0-1)^2\sigma^2-(\psi_0-1)(\alpha^O-\alpha)+\Delta_0\\ &= \left[\frac{\alpha^O-\alpha}{\sigma^2}+\Gamma_0\right]^2\sigma^2-\left[\frac{\alpha^O-\alpha}{\sigma^2}+\Gamma_0\right](\alpha^O-\alpha)+\Delta_0\\ &= \Gamma_0(\alpha^O-\alpha)+\Gamma_0^2\sigma^2+\Delta_0\end{aligned} \tag{B.42}$$

and

$$\sigma^{\eta}(0^{+})=(\psi_0-1)\sigma=\frac{\alpha^O-\alpha}{\sigma}+\Gamma_0\sigma. \tag{B.43}$$

For a net worth trap to be realized, we need to have $\tilde{D}_0\equiv\frac{2\mu^{\eta}(0^+)}{(\sigma^{\eta})^2(0^+)}<1$, i.e., $2\mu^{\eta}(0^+)<\sigma^{\eta}(0^+)^2$, which with equations (B.42) and (B.43) can be written as

$$2\left[\Gamma_0(\alpha^O-\alpha)+\Gamma_0^2\sigma^2+\Delta_0\right]<\left(\frac{\alpha^O-\alpha}{\sigma}+\Gamma_0\sigma\right)^2$$

which leads to

$$\left(\frac{\alpha^O-\alpha}{\sigma}\right)^2>\Gamma_0^2\sigma^2+2\Delta_0.$$

Therefore, when experts are optimistic, if $\alpha^O-\alpha>\sigma\sqrt{\Gamma_0^2\sigma^2+2\Delta_0}$, we have a net worth trap.

Pessimism Interestingly, we can see that if experts are pessimistic enough, i.e., if $\alpha^O-\alpha<-\sigma\sqrt{\Gamma_0^2\sigma^2+2\Delta_0}$, a net worth trap arises. The reason is different from the case of optimistic experts, however: pessimism of more productive users of capital (i.e., intermediaries) make them less willing to catch profitable (long) trades, even when the market risk premium is high enough. It can eventually drain their net worth in the long run. In sum, if

$$\left|\alpha^O-\alpha\right|>\sigma\sqrt{\Gamma_0^2\sigma^2+2\Delta_0}, \tag{B.44}$$

we have a net worth trap, regardless of the relative size of α^O and α.

Pessimism with short-sale constraints When α^O is low enough compared with α, i.e.,

$$\alpha^O-\alpha<-\sigma^2\left(1+\Gamma_0\right), \tag{B.45}$$

we can see from (B.40) that the asymptotic portfolio share of experts ψ_0 becomes negative. In that case, due to the short sale constraint of capital, $\psi_0 = 0$. In that case, we obtain

$$\sigma^\eta(0^+) = -\sigma, \ \ \mu^\eta(0^+) = \Delta_0 + (\sigma)^2 + \left(\alpha^O - \alpha\right).$$

Therefore, in order to have $\tilde{D}_0 < 1$, i.e., $\mu^\eta(0^+) < \frac{1}{2}\left(\sigma^\eta(0^+)\right)^2$, we need to have

$$\alpha^O - \alpha < -\left(\Delta_0 + \frac{1}{2}\sigma^2\right). \tag{B.46}$$

From (B.45) and (B.46), it should be

$$\alpha^O - \alpha < -\max\left\{\sigma^2\left(1+\Gamma_0\right), \Delta_0 + \frac{1}{2}\sigma^2\right\}. \tag{B.47}$$

Complete market In a complete market setting with $\delta^H = \delta^O$ and $l = 1$, i.e., households face no disadvantage in terms of the production efficiency, $\Gamma_0 = \Delta_0 = 0$. Thus, if $\alpha^O \neq \alpha$, the economy features a net worth trap and in the long run, households own the entire wealth. This result aligns with Blume and Easley (2006). In Online Appendix C, we study complete markets with a slightly different setting and show that our result that a small deviation from the rational expectations (e.g., $\alpha^O \neq \alpha$ in our model) leads to a net worth trap is robust across different model specifications.

■

Proof of Corollary 3. From (B.18), we obtain

$$\lim_{\eta\to 0} \sigma_t^p = \sigma.$$

From L'Hôpital's rule, we also obtain that:

$$\lim_{\eta\to 0} \sigma_t^\eta = \lim_{\eta\to 0} \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p = \lim_{\eta\to 0}\left(\psi_t'(\eta) - 1\right)\sigma = (\psi_0 - 1)\sigma \equiv \sigma^\eta(0^+).$$

where ψ_0 is defined in (B.38). As seen in (B.38), the derivative of ψ around $\eta \sim 0$, ψ_0, increases with optimism $\alpha^O - \alpha$, and, in the limit, the drift μ^η and volatility σ^η of η do not depend on η. Since now η in the limit (i.e., $\eta \sim 0$) follows a geometric Brownian motion with constant volatility $\sigma^\eta(0^+)$, its skewness is given by:

$$\left(\exp\left(\left(\sigma^\eta(0^+)\right)^2\right) + 2\right)\sqrt{\exp\left(\left(\sigma^\eta(0^+)\right)^2\right) - 1}$$

which is always positive, and increasing in $\sigma^{\eta}(0^{+})$. Therefore, as $\eta \to 0$, the skewness of the stationary distribution $d(\eta)$ increases when optimism $\alpha^{O} - \alpha$ increases.

■

B.3 Extension: with General Preferences

In this section, we prove the net worth trap result of Proposition 3 under more general utility functions of experts and households. In that purpose, we assume that experts maximize the following lifetime utility of consumption:

$$U_t^O = \sup_{C_t^O, x_t^O, \iota_t^O} \mathbb{E}_t^O \left[\int_t^\infty f(C_s^O, U_s^O) ds \right], \tag{B.48}$$

where

$$f(C, U) = \rho \cdot [(1-\gamma)U] \cdot \left(\log C - \frac{1}{1-\gamma} \log((1-\gamma)U) \right) \tag{B.49}$$

is a stochastic differential utility à la Duffie and Epstein (1992) with the elasticity of intertemporal substitution fixed at one. γ is the risk aversion. We assume that intermediaries and households both have γ as their relative risk aversion. If $\gamma = 1$, (B.48) becomes a usual log-utility case. ρ is the time discount rate for both intermediaries and households.

From (10), we define

$$dr_t^{Ok} = \underbrace{\left[\frac{1 - \iota_t^O}{q_t} + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p \right) + \frac{\alpha^O - \alpha}{\sigma} \sigma_t^p \right]}_{\equiv \widehat{r_t^{Ok}}} dt + \sigma_t^p dZ_t^O. \tag{B.50}$$

Then, experts with wealth W_t^O maximize (B.48) subject to the budget constraint:

$$\frac{dW_t^O}{W_t^O} = \underbrace{\left(r_t + x_t^O \left(\widehat{r_t^{Ok}} - r_t \right) - \tilde{c}_t^O \right)}_{\equiv \widehat{\mu_t^{W,O}}} dt + \underbrace{\left(x_t^O \sigma_t^p \right)}_{\equiv \sigma_t^{W,O}} dZ_t^O, \tag{B.51}$$

where $c_t^O = \tilde{c}_t^O W_t^O$, $x_t^O := \frac{p_t k_t^O}{W_t^O}$, and dZ_t^O is their (distorted) perceived Brownian motion.

Solution Using the preference in (B.48), we guess and verify the value function as:

$$U^j(\xi_t^j, W_t^j) = \frac{(W_t^j \xi_t^j)^{1-\gamma}}{1-\gamma},$$

for both intermediaries and households, i.e., $j = O, H$, where ξ_t^j depends on the aggregate state of the economy explained later. Its dynamics is conjectured to be

$$\frac{d\xi_t^j}{\xi_t^j} = \mu_t^{\xi,j} dt + \sigma_t^{\xi,j} dZ_t$$

where $\mu_t^{\xi,j}, \sigma_t^{\xi,j}$ will be determined in equilibrium. The expert's Hamilton-Jacobi-Bellman (HJB) equation is:

$$0 = \max_{x_t^O, \iota_t^O, \tilde{c}_t^O} \left\{ \frac{f(\tilde{c}_t^O W_t^O, U_t^O)}{(\xi_t^O W_t^O)^{1-\gamma}} + \mu^{\xi,O} + \widehat{\mu_t^{W,O}} - \frac{\gamma}{2}\left(\sigma_t^{W,O}\right)^2 - \frac{\gamma}{2}\left(\sigma_t^{\xi,O}\right)^2 + (1-\gamma)\sigma_t^{\xi,O}\sigma_t^{W,O} \right\},$$

where we use $(\xi_t^O W_t^O)^{1-\gamma} = (1-\gamma) U_t^O$, which can be rewritten as

$$\begin{aligned} 0 = \max_{x_t^O, \iota_t^O, \tilde{c}_t^O} \Big\{ &\rho \log(\tilde{c}_t^O) - \rho \log(\xi_t^O) + \mu_t^{\xi,O} + r_t - \tilde{c}_t^O + x_t^O \left(\widehat{r_t^{Ok}}(\iota_t^O) - r_t\right) - \frac{\gamma}{2}\left(x_t^O\right)^2 (\sigma_t^p)^2 \\ &- \frac{\gamma}{2}\left(\sigma_t^{\xi,O}\right)^2 + (1-\gamma) x_t^O \sigma_t^{\xi,O} \sigma_t^p \Big\} \end{aligned}$$

The first order conditions with respect to $\tilde{c}_t^O, \iota_t^O, x_t^O$ are

$$\tilde{c}_t^O = \rho \tag{B.52}$$

$$\Lambda'(\iota_t^O) = \frac{1}{q_t} \tag{B.53}$$

$$0 = (\widehat{r_t^{Ok}} - r_t) - \gamma x_t^O (\sigma_t^p)^2 + \underbrace{(1-\gamma)\sigma_t^{\xi,O}\sigma_t^p}_{\text{Hedging-demand term}}, \tag{B.54}$$

where the solution's form with $\gamma = 1$ (i.e., log-utility case) coincides with our Proposition 1. Note that with $\gamma > 1$, we have a hedging demand term as in Merton (1971).

The optimization problem and the first order conditions of agent h are identical except that as less productive households use the correct expectation, that is,

$$\tilde{c}_t^H = \rho \tag{B.55}$$

$$\Lambda'(\iota_t^H) = \frac{1}{q_t} \tag{B.56}$$

$$0 \geq (r_t^{Hk} - r_t) - \gamma x_t^H \left(\sigma_t^p\right)^2 + \underbrace{(1-\gamma)\sigma_t^{\xi,H}\sigma_t^p}_{\text{Hedging-demand term}}. \tag{B.57}$$

Dynamics of the state variable The process of η_t, the wealth share of experts, changes from the log-utility case. First, note that equations (B.9), (B.10), (B.11), (B.12) still hold. (B.13) changes to

$$\widehat{r_t^{Ok}} - r_t = r_t^{Ok} - r_t + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^p = \gamma x_t^O \left(\sigma_t^p\right)^2 - (1-\gamma)\sigma_t^{\xi,O}\sigma_t^p. \tag{B.58}$$

where r_t^{Ok} is the drift of the capital return process based on the true Brownian motion dZ_t. We use (B.58) to obtain

$$\begin{aligned}\frac{d\eta_t}{\eta_t} =& \frac{\psi_t - \eta_t}{\eta_t}\left(\gamma\frac{\psi_t}{\eta_t} - 1\right)(\sigma_t^p)^2 dt - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p dt + \frac{1-\iota_t^O}{q_t}dt + (1-\psi_t)\left(\delta^H - \delta^O\right)dt \\ &+ (1-l)(1-\psi_t)\Lambda^O\left(i_t^O\right)dt - \frac{\psi_t - \eta_t}{\eta_t}(1-\gamma)\sigma_t^{\xi,O}\sigma_t^p dt - \rho dt + \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p dZ_t.\end{aligned} \tag{B.59}$$

Also, (B.35) becomes

$$\begin{aligned}&\frac{(1-l)(1-\iota(q(\eta)))}{q(\eta)} + (1-l)\Lambda^O\left(\iota(q(\eta))\right) + \delta^H - \delta^O + \frac{\alpha^O - \alpha}{\sigma}\sigma^p(\eta) \\ &= \gamma\left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2 - (1-\gamma)\sigma^{\xi,O}(\eta)\sigma^p(\eta) + (1-\gamma)\sigma^{\xi,H}(\eta)\sigma^p(\eta).\end{aligned} \tag{B.60}$$

Deriving $\mu^\eta(0)$ and $\sigma^\eta(0)$ As in the log-utility case, we assume that as $\eta \to 0$, $q(\eta) \to q_0$. Let as assume $\psi(\eta) \simeq \psi_0\eta$ when $\eta \sim 0$. Since equations (B.19), (B.20), (B.21), (B.22), (B.23), (B.24), (B.25) hold, still $\sigma_t^p \to \sigma$, $\mu_t^p \to \alpha$, and $\sigma^\eta \to (\psi_0 - 1)\sigma$. Following steps are needed to calculate $\mu^\eta(0)$ and $\sigma^\eta(0)$:

Step 1 From (B.26), i.e., $\rho q_0 = l(1-\iota_o)$ we obtain q_0 and $\iota_0 = \iota(q_0) = \frac{q_0^2 - 1}{2k}$.

Step 2 For $j \in \{O, H\}$, $\xi_t^j = \xi^j(\eta_t)$ for some functions $\{\xi^j\}_{j\in\{O,H\}}$ in our Markov equilibrium. From

$$d\xi_t^j = \xi^{j\prime}(\eta_t)d\eta_t + \frac{1}{2}\xi^{j\prime\prime}(\eta_t)(d\eta_t)^2,$$

we obtain

$$\sigma_t^{\xi,j} = \frac{\xi^{j\prime}(\eta_t)}{\xi^j(\eta_t)} \cdot \eta_t \cdot \underbrace{\left(x_t^O - 1\right)\sigma_t^p}_{=\sigma_t^\eta} \tag{B.61}$$

which converges to 0 as η_t goes to 0, i.e., $\sigma_t^{\xi,j} \to 0$ for $j \in \{O, H\}$ as $\eta_t \to 0$. Thus, around $\eta_t \sim 0$, we need not consider the hedging demand terms in (B.59) and (B.60).

Step 3 From (B.59), we know that as $\eta \to 0$, $\mu^\eta(\eta) \to \mu^\eta(0)$, where $\mu^\eta(0)$ can be written as

$$\mu^\eta(0) = (\psi_0 - 1)(\gamma\psi_0 - 1)\sigma^2 - (\psi_0 - 1)(\alpha^O - \alpha) + \frac{1-\iota_0}{q_0} + (\delta^H - \delta^O) + (1-l)\Lambda^O(\iota_0) - \rho.$$

Step 4 From (B.29), we know

$$\frac{(1-l)(1-\iota_0)}{q_0} + (1-l)\Lambda^O(\iota_0) + (\delta^H - \delta^O) + \frac{\alpha^O - \alpha}{\cancel{\sigma}}\cancel{\sigma} = \gamma(\psi_0 - 1)\sigma^2 \tag{B.62}$$

as

$$\left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right) \to \psi_0 - 1.$$

Step 5 Therefore from the above 3 steps, we calculate μ_0 and ψ_0, from which we calculate $\sigma^\eta(0^+) = (\psi_0 - 1)\sigma$.

$$\mu^\eta(0) \equiv \lim_{\eta\to 0}\mu^\eta(\eta)$$
$$\sigma^\eta(0) \equiv \lim_{\eta\to 0}\sigma^\eta(\eta) = (\psi_0 - 1)\sigma.$$

Net worth trap condition With Γ_0 defined in (B.39), we obtain from Step 4 that $\psi_0 - 1 = \frac{1}{\gamma}\left(\frac{\alpha^O-\alpha}{\sigma^2} + \Gamma_0\right)$. From Step 3, with Δ_0 defined in (B.41), it follows that

$$\begin{aligned}\mu^\eta(0^+) &= (\psi_0 - 1)(\gamma\psi_0 - 1)\sigma^2 - (\psi_0 - 1)(\alpha^O - \alpha) + \Delta_0 \\ &= \frac{1}{\gamma}\left(\frac{\alpha^O - \alpha}{\sigma^2} + \Gamma_0\right)\left(\gamma - 1 + \frac{\alpha^O - \alpha}{\sigma^2} + \Gamma_0\right)\sigma^2 - \frac{1}{\gamma}\left[\frac{\alpha^O - \alpha}{\sigma^2} + \Gamma_0\right](\alpha^O - \alpha) + \Delta_0\end{aligned}$$

and

$$\sigma^\eta(0^+) = (\psi_0 - 1)\sigma = \frac{1}{\gamma}\left(\frac{\alpha^O - \alpha}{\sigma} + \Gamma_0\sigma\right).$$

For a net worth trap to be realized, we need to have $\tilde{D}_0 \equiv \frac{2\mu^\eta(0^+)}{(\sigma^\eta)^2(0^+)} < 1$, i.e., $2\mu^\eta(0^+) < \sigma^\eta(0^+)^2$. By defining

$$T_0 = \left\{\Gamma_0 - \gamma\left(\gamma - 1 + \Gamma_0\right)\right\}\sigma^2$$

and

$$P_0 = \sigma^2\left[\Gamma_0^2\sigma^2 - 2\gamma\left(\gamma - 1 + \Gamma_0\right)\Gamma_0\sigma^2 - 2\gamma^2\Delta_0\right]$$

Then, we should either have

$$\alpha^O - \alpha > -T_0 + \sqrt{T_0^2 - P_0},$$

i.e., optimism, or

$$\alpha^O - \alpha < -T_0 - \sqrt{T_0^2 - P_0},$$

i.e., pessimism.[7] Note that with $\gamma = 1$ (i.e., log-utility), the condition here coincides with the proof of Proposition 3.

Pessimism with short-sale constraints When α^O is low enough compared with α, i.e.,

$$\alpha^O - \alpha < -\sigma^2\left(\gamma + \Gamma_0\right), \tag{B.63}$$

we can see that the asymptotic portfolio share of experts ψ_0 becomes negative. In that case, due to the short sale constraint of capital, $\psi_0 = 0$. In that case, we obtain

$$\sigma^\eta(0^+) = -\sigma, \;\; \mu^\eta(0^+) = \Delta_0 + (\sigma)^2 + \left(\alpha^O - \alpha\right).$$

Therefore, in order to have $\tilde{D}_0 < 1$, i.e., $\mu^\eta(0^+) < \frac{1}{2}\left(\sigma^\eta(0^+)\right)^2$, we need to have

$$\alpha^O - \alpha < -\left(\Delta_0 + \frac{1}{2}\sigma^2\right). \tag{B.64}$$

[7]With $\gamma \geq 1$, $P_0 \leq 0$ so we always have a net worth trap under extreme optimism or pessimism.

From (B.63) and (B.64), it should be

$$\alpha^O - \alpha < -\max\left\{\sigma^2\left(\gamma + \Gamma_0\right), \Delta_0 + \frac{1}{2}\sigma^2\right\}. \tag{B.65}$$

B.4 The Household's Welfare Decomposition with Optimism

We compute the welfare change of the rational households due to optimism of experts in the economy as follows:

$$\text{Welfare Change} = \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log c_t^H dt\right] - \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log c_t^{H,REE} dt\right] \tag{B.66}$$

where $c_t^{H,REE}$ is the household's consumption in the rational expectation benchmark, i.e., when $\alpha^O = \alpha$. The welfare of the households can be decomposed as

$$\begin{aligned}\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log c_t^H dt\right] = & \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(1-\eta_t)dt\right]}_{\text{Wealth effect}_+} + \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(1-\iota_t)dt\right]}_{\text{Investment effect}_+} \\ & + \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log K_t dt\right]}_{\text{Capital effect}_-} + \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log A(\psi) dt\right]}_{\text{Misallocation effect}_-} + \underbrace{\text{t.i.e.}}_{\text{Terms independent of equilibria}}\end{aligned} \tag{B.67}$$

where $A(\psi) = \psi_t + l(1-\psi_t)$ is the aggregate (i.e., weighted average of capital share based on productivities) capital share in the economy, and $(1-\eta_t)$ captures the wealth share of the households. The wealth effect is an obvious direct effect that arises from a higher wealth share of households in the presence of optimism. Since the expectation errors particularly affect the experts by draining their net worth, the share of wealth held by households is higher. A higher occupancy time at the crisis due to optimism means that the investment rate is lower, leaving the households to consume more. However, there are other general equilibrium effects that arise out of a higher occupancy time in a crisis due to optimism. First, the aggregate capital stock will be lower in the future as well due to a lower growth rate of capital. Similarly, the share of capital held by households, who are less productive than experts, gets bigger, captured by the misallocation effect. The last two components have a negative effect on the welfare. We perform numerical simulations and show that the negative general equilibrium effects dominate the direct wealth and investment effect, leading to an overall decline in welfare. In fact, all these forces become stronger as optimism increases, and thus the total welfare of rational agents decrease in optimism as displayed in Figure A2, where we use the slightly different calibration:[8]

	l	δ^O	δ^H	ρ	σ	k	α
Values	0.4	0	0	0.03	0.08	18	0.07

Table A1: Parameterization for $\alpha^O \geq \alpha$ for welfare calculation

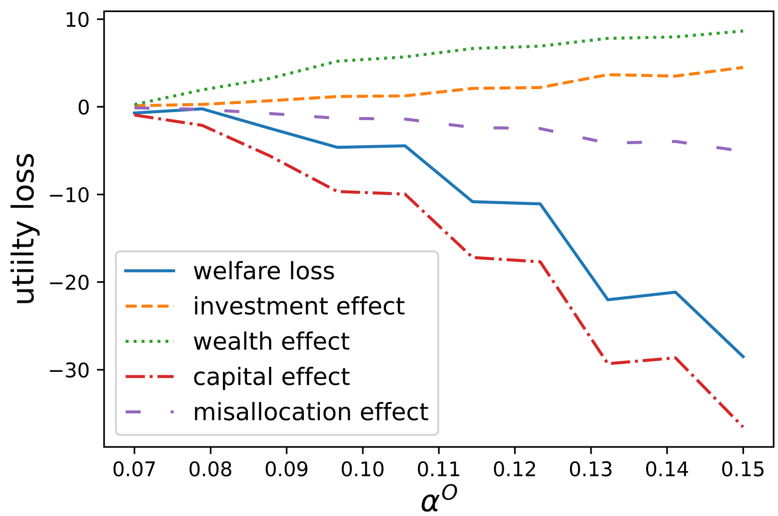


Figure A2: Decomposition of the welfare loss of households

B.4.1 Details of Calculations

From the goods market clearing condition (B.28), i.e., $k_t^O\left(\gamma_t^O - \iota_t^O\gamma_t^O\right) + k_t^P\left(\gamma_t^H - \iota_t^H\gamma_t^H\right) = C_t$, where C_t is aggregate consumption, and $\frac{C_t}{W_t} = \frac{c_t^H}{W_t^H} = \rho$,[9] we obtain $c_t^h = \rho(1-\eta_t)W_t = (1-\eta_t)K_t\gamma_t^O \cdot (1-\iota_t)A(\psi_t)$. Then we obtain the decomposition of welfare in (B.67).

For the capital effect, we obtain

$$\underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log K_t dt\right]}_{\text{Capital effect}} = \frac{\log K_0}{\rho} + \frac{1}{\rho}\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\left(\Lambda^O\left(i_t^O\right) - \delta^O - (1-\psi_t)(1-l)\Lambda^O\left(i_t^O\right)\right)dt\right].$$

[8]Our calibration in Section 2.4 features a very strong effect of belief distortions on the household welfare, so we change the calibration to illustrate how each effect in (B.67) changes with the degree of optimism.

[9]W_t is the aggregate wealth of the economy, i.e., p_tK_t.

For the wealth effect, from (27) we obtain

$$d\log(1-\eta_t) = \left(-\frac{\eta_t}{1-\eta_t}\mu_t^\eta + \frac{1}{2}\frac{\eta_t^2}{(1-\eta_t)^2}(\sigma_t^\eta)^2\right)dt - \frac{\eta_t}{1-\eta_t}\sigma_t^\eta dZ_t.$$

Therefore,

$$\underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(1-\eta_t)dt\right]}_{\text{Wealth effect}} = \frac{\log(1-\eta_0)}{\rho} + \frac{1}{\rho}\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\left(-\frac{\eta_t}{1-\eta_t}\mu_t^\eta + \frac{1}{2}\left(\frac{\eta_t}{1-\eta_t}\right)^2(\sigma_t^\eta)^2\right)dt\right]$$

where under $\delta^H = \delta^O = 0$, we use

$$\mu_t^\eta = \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p + \frac{1-\iota_t^O}{q_t} + (1-l)\,(1-\psi_t)\,\Lambda^O(i_t^O) - \rho,$$

and

$$\sigma_t^\eta = \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p.$$

Investment and misallocation effects can be calculated based on simulations.

Online Appendix for
Beliefs and the Net Worth Trap

GOUTHAM GOPALAKRISHNA SEUNG JOO LEE THEOFANIS PAPAMICHALIS

Appendix C General Models with Complete Markets

In Online Appendix, we illustrate that the apparition of net worth trap is not specific to our model specification or the unit risk aversion (i.e., log-preference) of optimists and households. In that purpose, based on a slightly simpler model specification of behavioral biases and recursive utility à la Duffie and Epstein (1992), we prove that a net worth trap still appears. Online Appendix C focuses on complete markets, while Online Appendix D focuses on the incomplete market setting as in our main body of the paper.

Setting There are two types of agents in the economy indexed by $j \in i, h$. i is intermediary (or expert, interchangeably), while h is household. There is one capital in the economy that evolves as[10]

$$dk_t^a = k_t^a \left(\left(\mu^a + \Phi(\iota_t^a) \right) dt + \sigma^a dZ_t^a \right) \tag{C.1}$$

Expert i is optimistic and believes that the growth rate of capital is $\mu^O > \mu^a$, in a similar way to Basak (2000) and our Section 2. Later, we consider pessimism cases ($\mu^O < \mu^a$) as well. Thus, the perceived capital process of the agent is given by

$$dk_t^a = k_t^a \left(\left(\mu^O + \Phi(\iota_t^a) \right) dt + \sigma^a \widehat{dZ_t^a} \right) \tag{C.2}$$

where

$$\widehat{dZ_t^a} := dZ_t^a - \frac{\mu^O - \mu^a}{\sigma^a} dt.$$

is the Brownian motion retrieved by intermediaries. The price of the capital is given by

$$\frac{dq_t^a}{q_t^a} = \mu_t^{qa} dt + \sigma_t^{qa,a} dZ_t^a \tag{C.3}$$

[10] Brownian motion dZ_t^a is a fundamental source of variation in this version of the model, as there is no technological growth.

where both μ_t^{qa} and $\sigma_t^{qa,a}$ are endogenous. Then, the stochastic process of capital price q_t^a perceived by experts can be written as

$$\frac{dq_t^a}{q_t^a} = \left(\mu_t^{qa} + \frac{\mu^O - \mu^a}{\sigma^a}\sigma_t^{qa,a}\right) dt + \sigma_t^{qa,a}\widehat{dZ_t^a}. \tag{C.4}$$

In contrast to our main model in Section 2, there is no technological growth and the TFP is constant. In our complete market setting, we assume that capital k_t^a produces $y_t^a = \alpha^a k_t^a$ for both intermediaries i and households h,[11] so the actual return process for intermediary i is computed as

$$dr_t^{i,ka} = \frac{d(q_t k_t^a)}{q_t k_t^a} + \underbrace{\frac{\alpha^a - \iota_t^a}{q_t^a}dt}_{\text{Dividend yield}} = \underbrace{\left(\mu_t^{qa} + \mu^a + \Phi(\iota_t^a) + \sigma^a\sigma_t^{qa,a} + \frac{\alpha^a - \iota_t^a}{q_t^a}\right)}_{\equiv r_t^{i,ka}} dt + (\sigma^a + \sigma_t^{qa,a})dZ_t^a$$

where $\iota_t^a k_t^a$ is the amount of investment in physical capital. The agent i uses the perceived capital process (C.2) and the perceived price process (C.4) to compute the return process as

$$\begin{aligned} dr_t^{i,ka} &= \frac{d(q_t k_t^a)}{q_t k_t^a} + \frac{\alpha^a - \iota_t^a}{q_t^a}dt \\ &= \underbrace{\left(\mu_t^{qa} + \mu^a + \Phi(\iota_t^a) + \sigma^a\sigma_t^{qa,a} + \frac{\alpha^a - \iota_t^a}{q_t^a} + \frac{\mu^O - \mu^a}{\sigma^a}(\sigma^a + \sigma_t^{qa,a})\right)}_{\equiv \widehat{r_t^{i,ka}}} dt + (\sigma^a + \sigma_t^{qa,a})\widehat{dZ_t^a}. \end{aligned}$$

The actual expected return on capital for rational household h is given by

$$\begin{aligned} dr_t^{h,ka} &= \frac{d(q_t^a k_t^a)}{q_t^a k_t^a} + \frac{\alpha^a - \iota_t^a}{q_t^a}dt \\ &= \underbrace{\left(\mu_t^{qa} + \mu^a + \Phi(\iota_t^a) + \sigma^a\sigma_t^{qa,a} + \frac{\alpha^a - \iota_t^a}{q_t^a}\right)}_{\equiv r_t^{h,ka}} dt + (\sigma^a + \sigma_t^{qa,a})dZ_t. \end{aligned}$$

The household h is rational and thus uses the correct return processes (C.1) and (C.3).

[11] Thus, $\ell = 1$ in our complete market setting.

Preferences The problem of optimist i is to maximize her lifetime utility of consumption

$$U_t^i = \sup_{C_t^i, w_t^{i,a}, \iota_t^a} \widehat{\mathbb{E}}_t \left[\int_t^\infty f(C_s^i, U_s^i) ds \right], \tag{C.5}$$

where

$$f(C, U) = \rho \cdot [(1-\gamma)U] \cdot \left(\log C - \frac{1}{1-\gamma} \log((1-\gamma)U) \right) \tag{C.6}$$

is a stochastic differential utility à la Duffie and Epstein (1992) with the elasticity of intertemporal substitution fixed at one. γ is the risk aversion. We assume that intermediaries and households both have γ as their relative risk aversion. If $\gamma = 1$, (C.5) becomes a usual log-utility case. ρ is the time discount rate for both intermediaries and households.

Thus, optimists maximize (C.5) satisfying the following budget constraint:

$$\frac{dn_t^i}{n_t^i} = \underbrace{\left(r_t + w_t^{i,a} \left(\widehat{r_t^{i,ka}} - r_t \right) - c_t^i \right)}_{\equiv \widehat{\mu_t^{ni}}} dt + \underbrace{\left(w_t^{i,a} \left(\sigma^a + \sigma_t^{qa,a} \right) \right)}_{\equiv \sigma_t^{ni,a}} \widehat{dZ_t^a}, \tag{C.7}$$

where $C_t^i = c_t^i n_t^i$ and $w_t^{i,a} := \frac{q_t^a k_t^a}{n_t^i}$ The actual dynamics with correct Brownian motion dZ_t^a will feature the mean of (C.7) as

$$\mu^{ni} = \left(1 - w_t^{i,a}\right) r_t + w_t^{ia} r_t^{i,ka} - c_t^i.$$

Solution Using the preference in (C.6), following the literature including Duffie and Epstein (1992), we guess and verify the value function as:

$$U^j(\xi_t^j, n_t^j) = \frac{(n_t^j \xi_t^j)^{1-\gamma}}{1-\gamma},$$

for both intermediaries and households, i.e., $j = i, h$, where ξ_t^j depends on the aggregate state of the economy explained later. Its dynamics is conjectured to be

$$\frac{d\xi_t^i}{\xi_t^i} = \mu_t^{\xi,i} dt + \sigma_t^{\xi i,a} dZ_t^a$$

where $\mu_t^{\xi,i}, \sigma_t^{\xi i,a}$ will be determined in equilibrium. We can write the optimist's Hamilton-Jacobi-Bellman (HJB) equation as:

$$0 = \max_{w_t^{i,a}, \iota_t^a, c_t^i} \left\{ \frac{f(c_t^i n_t^i, U_t^i)}{(\xi_t^i n_t^i)^{1-\gamma}} + \mu^{\xi i} + \widehat{\mu_t^{ni}} - \frac{\gamma}{2}(\sigma_t^{ni,a})^2 - \frac{\gamma}{2}(\sigma_t^{\xi i,a})^2 + (1-\gamma)\sigma_t^{\xi i,a}\sigma_t^{ni,a} \right\},$$

where we use $(\xi_t^i n_t^i)^{1-\gamma} = (1-\gamma)U_t^i$, which can be rewritten as

$$0 = \max_{w_t^{i,a}, \iota_t^a, c_t^i} \left\{ \rho \log(c_t^i) - \rho \log(\xi_t^i) + \mu_t^{\xi,i} + r_t - c_t^i + w_t^{i,a} \left(\widehat{r_t^{ka}}(\iota_t^a) - r_t \right) - \frac{\gamma}{2} \left(w_t^{i,a} \right)^2 (\sigma^a + \sigma_t^{qa,a})^2 \right.$$
$$\left. - \frac{\gamma}{2} \left(\sigma_t^{\xi,i,a} \right)^2 + (1-\gamma) w_t^{i,a} \sigma_t^{\xi,i,a} (\sigma^a + \sigma_t^{qa,a}) \right\}$$

The first order conditions with respect to $c_t^i, \iota_t^a, w_t^{i,a}$ are

$$c_t^i = \rho \tag{C.8}$$
$$\Phi'(\iota_t^a) = \frac{1}{q_t^a} \tag{C.9}$$
$$0 = (\widehat{r_t^{ka}} - r_t) - \gamma w_t^{i,a} (\sigma^a + \sigma_t^{qa,a})^2 + \underbrace{(1-\gamma)\sigma_t^{i,\xi,a}(\sigma^a + \sigma_t^{qa,a}))}_{\text{Hedging term}} \tag{C.10}$$

where the solution's form with $\gamma = 1$ (i.e., log-utility case) coincides with our Proposition 1. Note that with $\gamma > 1$, we have a hedging demand term as in Merton (1971).

The optimization problem and the first order conditions of agent h are identical except that as less productive household h uses the correct expectation, i.e., $r_t^{h,ka}$ is used. That is,

$$c_t^h = \rho \tag{C.11}$$
$$\Phi'(\iota_t^a) = \frac{1}{q_t^a} \tag{C.12}$$
$$0 \geq (r_t^{ka} - r_t) - \gamma w_t^{h,a} (\sigma^a + \sigma_t^{qa,a})^2 + (1-\gamma)\sigma_t^{h,\xi,a} (\sigma^a + \sigma_t^{qa,a}), \tag{C.13}$$

Market clearing We have the following market clearing conditions written in $\eta_t := \frac{n_t^i}{n_t}$, which denotes the share of wealth held by i, with the fact that both types $j = i, h$ choose the same investment ratio ι_t^a as confirmed in (C.9) and (C.12):

$$\left(c_t^i \eta_t + c_t^h (1-\eta_t) \right) q_t = \underbrace{(w_t^{ia}\eta_t + w_t^{ha}(1-\eta_t))}_{=1} \alpha^a - \iota_t^a, \tag{C.14}$$

with $k_t = k_t^i + k_t^h$. The capital market clearing condition is given by

$$w_t^{ia}\eta_t + w_t^{ha}(1-\eta_t) = 1. \tag{C.15}$$

Markov equilibrium: As we do in Section 2.4, we can construct a Markov equilibrium in one state variable, η_t. We assume that each type $j = i, h$'s net worth n_t^j follows

$$\frac{dn_t^j}{n_t^j} = \mu_t^{nj}dt + \sigma_t^{nj,a}dZ_t^a,$$

where both μ_t^{nj} and $\sigma_t^{nj,a}$ are endogenous. We then describe the dynamics of the economy's total wealth n_t as follows:

$$\begin{aligned} dn_t &= dn_t^i + dn_t^h \\ &= \underbrace{\left(\eta_t\mu_t^{ni} + (1-\eta_t)\mu_t^{nh}\right)}_{\equiv\mu_t^n} n_t dt + \underbrace{\left(\eta_t\sigma_t^{ni,a} + (1-\eta_t)\sigma_t^{nh,a}\right)}_{\equiv\sigma_t^{n,a}} n_t dZ_t^a. \end{aligned} \tag{C.16}$$

And we obtain

$$\begin{aligned} \frac{d\eta_t}{\eta_t} &= \frac{dn_t^i}{n_t^i} - \frac{dn_t}{n_t} + \left(\frac{dn_t}{n_t}\right)^2 - \frac{dn_t^i}{n_t^i}\cdot\frac{dn_t}{n_t} \\ &= \left((1-\eta_t)(\mu_t^{n,i} - \mu_t^{n,h}) + (\sigma_t^{n,a})^2 - \sigma_t^{ni,a}\sigma_t^{n,a}\right)dt + (1-\eta_t)\left(\sigma_t^{ni,a} - \sigma_t^{nh,a}\right)dZ_t^a. \end{aligned}$$

A more intuitive way of rewriting the dynamics of η_t is as follows. From $\eta_t = \frac{n_t^i}{q_t^a k_t^a}$, we obtain

$$\frac{d(q_t^a k_t^a)}{q_t^a k_t^a} = \underbrace{\left(\mu^a + \Phi(\iota_t^a) + \mu_t^{qa} + \sigma^a\sigma_t^{qa,a}\right)}_{\equiv\mu_t^{qk}} dt + (\sigma^a + \sigma_t^{qa,a})dZ_t^a,$$

which leads to

$$\begin{aligned} \frac{d\eta_t}{\eta_t} &= \frac{dn_t^i}{n_t^i} - \frac{d(q_t^a k_t^a)}{q_t^a k_t^a} + \left(\frac{d(q_t^a k_t^a)}{q_t^a k_t^a}\right)^2 - \frac{dn_t^i}{n_t^i}\cdot\frac{d(q_t^a k_t^a)}{q_t^a k_t^a} \\ &= \left(r_t + w_t^{i,a}\left(\underbrace{r_t^{i,ka}}_{\text{True expected return}} - r\right) - c_t^i - \mu_t^{qk} + (\sigma^a + \sigma_t^{qa,a})^2\left(1 - w_t^{i,a}\right)\right)dt + (w_t^{ia} - 1)(\sigma^a + \sigma_t^{qa,a})dZ_t^a. \end{aligned}$$

C.1 With $\gamma = 1$ (i.e., log-utility)

In this case, the hedging demand terms in (C.10) and (C.13) disappear. From equation (C.10), we have

$$w_t^{i,a} = \frac{\widehat{r_t^{i,ka}} - r_t}{(\sigma^a + \sigma_t^{qa,a})^2} = \frac{r_t^{i,ka} - r_t}{(\sigma^a + \sigma_t^{qa,a})^2} + \frac{\frac{\mu^O - \mu^a}{\sigma^a}(\sigma^a + \sigma_t^{qa,a})}{(\sigma^a + \sigma_t^{qa,a})^2}$$

leading to

$$r_t^{i,ka} - r_t = w_t^{i,a}\,(\sigma^a + \sigma_t^{qa,a})^2 - \frac{\mu^O - \mu^a}{\sigma^a}\,(\sigma^a + \sigma_t^{qa,a}) \tag{C.17}$$

Plugging (C.17) in the drift of η_t above, we obtain

$$\mu_t^\eta = r_t + \left(w_t^{i,a}(\sigma^a + \sigma_t^{qa,a})\right)^2 - \underbrace{w_t^{i,a}\frac{\mu^O - \mu^a}{\sigma^a}\,(\sigma^a + \sigma_t^{qa,a})}_{\text{Expectation error}} + (\sigma^a + \sigma_t^{qa,a})^2\,(1 - w_t^{i,a}) - c_t^i - \mu_t^{qk} \tag{C.18}$$

which contains a term stemming from the expectation error of optimistic intermediaries i. For $\mu^O > \mu^a$ case, the error comes from the fact that the agent takes portfolio decisions based on a higher perceived risk premium, but gains a lower risk premium.

Note that we have

$$\begin{aligned} r_t - \mu_t^{qk} &= \frac{\alpha^a - \iota_t^a}{q_t^a} - \left(r_t^{i,ka} - r_t\right) \\ &= \frac{\alpha^a - \iota_t^a}{q_t^a} - w_t^{i,a}\,(\sigma^a + \sigma_t^{qa,a})^2 + \frac{\mu^O - \mu^a}{\sigma^a}\,(\sigma^a + \sigma_t^{qa,a}) \end{aligned}$$

Substituting in the drift of wealth share (C.18), we obtain

$$\mu_t^\eta = \frac{\alpha^a - \iota_t^a}{q_t^a} + \left((w_t^{i,a} - 1)(\sigma^a + \sigma_t^{qa,a})\right)^2 - (w_t^{i,a} - 1)\frac{\mu^O - \mu^a}{\sigma^a}\,(\sigma^a + \sigma_t^{qa,a}) - \rho,$$

with

$$\sigma_t^\eta = (w_t^{i,a} - 1)(\sigma^a + \sigma_t^{qa,a}). \tag{C.19}$$

Ergodic distribution and net worth trap As proven in Section B.2, the asymptotic distribution of our state variable η_t when $\eta_t \sim 0$ is given by

$$d(\eta) \sim \left(\frac{2\mu_0^\eta}{(\sigma_0^\eta)^2} - 1\right)\eta^{\frac{2\mu_0^\eta}{(\sigma_0^\eta)^2} - 2} \tag{C.20}$$

where the ratio $\tilde{D}_0 := \frac{2\mu_0^\eta}{(\sigma_0^\eta)^2}$ determines the existence of a degenerate distribution around $\eta = 0$. If $\tilde{D}_0 < 2$, the density features an infinite mass around $\eta_t = 0$. If $\tilde{D}_0 < 1$, the density becomes degenerate at $\eta_t = 0$, i.e., the economy features the net worth trap.

In our Markov equilibrium, $q_t^a = q(\eta_t)$ for some function $q(\cdot)$. Then

$$dq_t^a = q'(\eta_t)d\eta_t + \frac{1}{2}q''(\eta_t)\left(d\eta_t\right)^2,$$

leading to

$$\sigma_t^{qa,a} = \frac{q'(\eta_t)}{q(\eta_t)}\eta_t \cdot \left(w_t^{i,a} - 1\right)\left(\sigma^a + \sigma_t^{qa,a}\right), \tag{C.21}$$

where $\sigma_t^{qa,a} \to 0$ as $\eta_t \to 0$.

Around $\eta \sim 0$ At the limit $\eta_t \to 0^+$, it will be that $w_t^{h,a} \to 1$ from (C.15), from which we obtain[12]

$$q_0^a = \frac{1 + \alpha^a\kappa}{1 + \rho\kappa}, \quad \iota_0^a = \frac{q_0 - 1}{\kappa} = \frac{\alpha^a - \rho}{1 + \rho\kappa}, \quad \sigma_0^{qa,a} = 0,$$

leading to

$$w_0^{i,a} = \frac{\mu^O - \mu^a}{(\sigma^a)^2} + 1 \tag{C.22}$$

$$\mu_0^\eta = \cancel{\frac{\alpha^a - \iota_0}{q_0}} - \rho + \left((w_0^{ia} - 1)\sigma^a\right)^2 - \frac{\mu^O - \mu^a}{\cancel{\sigma^a}}(w_0^{ia} - 1)\cancel{\sigma_a} = 0 \tag{C.23}$$

$$\sigma_0^\eta = (w_0^{ia} - 1)\sigma^a = \frac{\mu^O - \mu^a}{\sigma^a} \tag{C.24}$$

which leads to $\tilde{D}_0 = 0$, which is clearly below 1. Thus, we have the following results:

Case.1 In the optimism case ($\mu^O > \mu^a$), we have a net worth trap.

Case.2 In the pessimism case ($\mu^O < \mu^a$), a net worth trap arises. Therefore, only with slight deviations from the rational expectations, we can have a net worth trap.

It can be understood more in detail as follows.

Observation We calculate that for $\forall t$, from $w_t^{i,a}\eta_t + w_t^{h,a}(1 - \eta_t) = 1$,

$$w_t^{i,a} - 1 = \frac{\mu^O - \mu^a}{(\sigma^a)^2}(1 - \eta_t) \tag{C.25}$$

[12]We put subscript 0 to denote that variables are valued at $\eta \to 0^+$ limit.

leading to

$$
\begin{aligned}
\mu_t^{\eta} &= \left((w_t^{i,a}-1)\sigma^a\right)^2 - (w_t^{i,a}-1)\frac{\mu^O-\mu^a}{\sigma^a}\sigma^a = (w_t^{i,a}-1)\sigma^a\left[\frac{\mu^O-\mu^a}{\sigma^a}(\cancel{1}-\eta_t) - \cancel{\frac{\mu^O-\mu^a}{\sigma^a}}\right] \\
&= -\eta_t(1-\eta_t)\left(\frac{\mu^O-\mu^a}{\sigma^a}\right)^2
\end{aligned}
\tag{C.26}
$$

which is always negative for $0<\eta_t<1$. Therefore, regardless of $\mu^O>\mu^a$ (i.e., optimism) or $\mu^O<\mu^a$ (i.e., pessimism), the distribution is degenerate at $\eta_t=0$. This result aligns with Blume and Easley (2006).

C.2 With $\gamma>1$ (i.e., non log-utility)

In the case $\gamma>1$, the hedging terms also appear in the drift, as seen in (C.10) and (C.13). That is, we have

$$
\begin{aligned}
\widehat{r_t^{ka}} - r_t &= r_t^{ka} - r_t + \frac{\mu^O-\mu^a}{\sigma^a}(\sigma^a+\sigma_t^{qa,a}) \\
&= \gamma w_t^{i,a}\left(\sigma^a+\sigma_t^{qa,a}\right)^2 - \underbrace{(1-\gamma)\sigma_t^{i,\xi,a}(\sigma^a+\sigma_t^{qa,a})}_{\text{Hedging demand term}}.
\end{aligned}
\tag{C.27}
$$

If we plug in (C.27) to the original μ_t^{η} equation (C.18), we obtain

$$
\begin{aligned}
\mu_t^{\eta} &= r_t + w_t^{i,a}\left(r_t^{ka}-r\right) - c_t^i - \mu_t^{qk} + (\sigma^a+\sigma_t^{qa,a})^2\left(1-w_t^{i,a}\right) \\
&= r_t + (w_t^{i,a})^2\left((\sigma^a+\sigma_t^{qa,a})^2\gamma\right) - w_t^{i,a}\frac{\alpha^O-\alpha^a}{\sigma^a}(\sigma^a+\sigma_t^{qa,a}) \\
&\quad - w_t^{i,a}(1-\gamma)\sigma_t^{i,\xi_i,a}(\sigma^a+\sigma_t^{qa,a}) + (\sigma^a+\sigma_t^{qa,a})^2(1-w_t^{i,a}) - c_t^i - \mu_t^{qk}.
\end{aligned}
\tag{C.28}
$$

Then, we obtain

$$
\begin{aligned}
r_t - \mu_t^{qk} &= \frac{\alpha^a-\iota_t^a}{q_t^a} - \left(r_t^{ka}-r_t\right) \\
&= \frac{\alpha^a-\iota_t^a}{q_t^a} - \gamma w_t^{i,a}(\sigma^a+\sigma_t^{qa,a})^2 + \frac{\mu^O-\mu^a}{\sigma^a}(\sigma^a+\sigma_t^{qa,a}) + (1-\gamma)\sigma_t^{i,\xi,a}(\sigma^a+\sigma_t^{qa,a})
\end{aligned}
\tag{C.29}
$$

Substituting (C.29) in the drift of wealth share (C.28), we obtain

$$\mu_t^\eta = \cancel{\frac{\alpha^a - \iota_t^a}{q_t^a}} + \gamma\left((w_t^{i,a}-1)(\sigma^a+\sigma_t^{qa,a})\right)^2 - (w_t^{i,a}-1)\frac{\mu^O-\mu^a}{\sigma^a}(\sigma^a+\sigma_t^{qa,a}) - \cancel{\rho}$$
$$+ (1-\gamma)(\sigma^a+\sigma_t^{qa,a})^2\left(1-w_t^{i,a}\right) + \left(1-w_t^{i,a}\right)(1-\gamma)\sigma_t^{i,\xi,a}(\sigma^a+\sigma_t^{qa,a}). \tag{C.30}$$

For $j \in \{i,h\}$, $\xi_t^j = \xi^j(\eta_t)$ for some functions $\{\xi^j\}_{j\in\{i,h\}}$ in our Markov equilibrium. From

$$d\xi_t^j = \xi^{j\prime}(\eta_t)d\eta_t + \frac{1}{2}\xi^{j\prime\prime}(\eta_t)(d\eta_t)^2,$$

we obtain

$$\sigma_t^{j,\xi,a} = \frac{\xi^{j\prime}(\eta_t)}{\xi^j(\eta_t)}\cdot\eta_t\cdot\underbrace{\left(w_t^{ia}-1\right)(\sigma^a+\sigma_t^{qa,a})}_{=\sigma_t^\eta} \tag{C.31}$$

which converges to 0 as η_t goes to 0, i.e., $\sigma_t^{j,\xi,a} \to 0$ for $j \in \{i,h\}$ as $\eta_t \to 0$. Thus, around $\eta_t \sim 0$, we need not consider the hedging demand term. From the good market equilibrium condition (C.15), we obtain

$$q_t^a = \frac{1+\alpha^a\kappa}{1+\rho\kappa}, \quad \iota_t = \frac{q_t^a - 1}{\kappa}.$$

implying $\sigma_t^{qa,a} = 0$ for $\forall t$ as well. From (C.15), we have: $1 = w_t^{i,a}\eta_t + w_t^{h,a}(1-\eta_t)$, where we obtain $w_t^{h,a} \to 1$ when $\eta_t \to 0$, which implies

$$r_t^{ka} - r_t = \gamma\left(\sigma^a + \underbrace{\sigma_t^{qa,a}}_{=0}\right)^2 - (1-\gamma)\underbrace{\sigma_t^{h,\xi,a}}_{\to 0}(\sigma^a+\sigma_t^{qa,a}),$$

leading to

$$w_t^{i,a} = \frac{\mu^O-\mu^a}{\gamma(\sigma^a)^2} + 1.$$

Therefore, in the limit $\eta_t \to 0^+$, we have the followings:

$$q_0 = \frac{1+\alpha^a\kappa}{1+\rho\kappa}, \quad \iota_0 = \frac{q_0-1}{\kappa}, \quad {\sigma_0}^{qa,a} = 0, \quad w_0^{ia} = \frac{\mu^O-\mu^a}{\gamma(\sigma^a)^2} + 1 \tag{C.32}$$

$$\mu_0^\eta = \gamma\left((w_0^{i,a}-1)\sigma^a\right)^2 - \frac{\mu^O-\mu^a}{\cancel{\sigma^a}}(w_0^{i,a}-1)\cancel{\sigma^a} + (1-\gamma)(\sigma^a)^2(1-w_0^{i,a}) \tag{C.33}$$

$$\sigma_0^\eta = (w_0^{i,a}-1)\sigma^a \tag{C.34}$$

A net worth trap around $\eta_t \sim 0$ can appear when $\tilde{D}_0 < 1$, i.e., $\mu_0^\eta < \frac{1}{2}\left(\sigma_0^\eta\right)^2$ where μ_0^η and σ_0^η are from (C.33) and (C.34). It can be written as

$$\left(\mu^O - \mu^a\right)\left(1 - w_0^{i,a}\right) + (1-\gamma)(\sigma^a)^2\left(1 - w_0^{i,a}\right) < \left(\frac{1}{2} - \gamma\right)(\sigma^a)^2\left(1 - w_0^{i,a}\right)^2 \tag{C.35}$$

Case.1 If $w_0^{i,a} < 1$, then (C.35) becomes

$$\mu^O - \mu^a + (1-\gamma)(\sigma^a)^2 < \left(\frac{1}{2} - \gamma\right)(\sigma^a)^2\left(1 - w_0^{i,a}\right)$$

becoming

$$\begin{aligned}
\mu^O - \mu^a &< -\left(\frac{1}{2} - \gamma\right)(\sigma^a)^2 w_0^{i,a} - \frac{1}{2}(\sigma^a)^2 \\
&= -\left(\frac{1}{2} - \gamma\right)(\sigma^a)^2\left[\frac{\mu^O - \mu^a}{\gamma(\sigma^a)^2} + 1\right] - \frac{1}{2}(\sigma^a)^2 \\
&= -\frac{1-2\gamma}{2\gamma}\left(\mu^O - \mu^a\right) - \left(\frac{1}{2} - \gamma\right)(\sigma^a)^2 - \frac{1}{2}(\sigma^a)^2
\end{aligned} \tag{C.36}$$

leading to

$$\frac{\mu^O - \mu^a}{2\gamma} < -\left(\frac{1}{2} - \gamma\right)(\sigma^a)^2 - \frac{1}{2}(\sigma^a)^2$$

which can be written as

$$\begin{aligned}
\mu^O - \mu^a &< -(1-2\gamma)\gamma(\sigma^a)^2 - \gamma(\sigma^a)^2 \\
&= 2\gamma(\gamma - 1)(\sigma^a)^2.
\end{aligned} \tag{C.37}$$

Of course, we need to have $w_0^{i,a} < 1$ which is written as

$$\frac{\mu^O - \mu^a}{\gamma(\sigma^a)^2} + 1 < 1$$

leading to

$$\mu^O - \mu^a < 0. \tag{C.38}$$

If $\gamma \geq 1$, then (C.38) implies (C.37). Therefore, If (C.38) is satisfied, we have $\tilde{D}_0 < 1$, i.e., net worth trap around $\eta_t \sim 0$. Thus, a small degree of pessimism leads to a net worth trap as in Section C.1.

Case.2 If $w_0^{i,a} > 1$, then (C.35) becomes

$$\mu^O - \mu^a + (1-\gamma)(\sigma^a)^2 > \left(\frac{1}{2} - \gamma\right)(\sigma^a)^2\left(1 - w_0^{i,a}\right)$$

leading to

$$\mu^O - \mu^a > 2\gamma(\gamma - 1)(\sigma^a)^2. \tag{C.39}$$

Since if (C.39) is satisfied, with $\gamma \geq 1$ we have $w_0^{i,a} > 1$, i.e.,

$$\mu^O - \mu^a > 0,$$

(C.39) is sufficient for $\tilde{D}_0 < 1$, i.e., a net worth trap at $\eta_t \sim 0$. The optimism degree $\mu^O - \mu^a$ must be high enough to satisfy (C.39) to generate a net worth trap, in contrast to Section C.1.

Appendix D General Models with Incomplete Markets

Based on a similar framework to Online Appendix C, this section focuses on the incomplete market setting and illustrates that the concept of net worth trap is not specific to our model specification or the utility function of intermediaries and households.

There are two types of agents in the economy indexed by $j \in i, h$. i is intermediary (or expert), while h is household. There is one capital in the economy that evolves as

$$dk_t^a = k_t^a \left((\mu^a + \Phi(\iota_t^a)) \, dt + \sigma^a dZ_t^a \right) \tag{D.1}$$

Expert i is optimistic and believes that the growth rate of capital is $\mu^O > \mu^a$, in a similar way to Basak (2000) and our Section 2. Later, we consider pessimism cases ($\mu^O < \mu^a$) as well. Thus, the perceived capital process of the agent is given by

$$dk_t^a = k_t^a \left(\left(\mu^O + \Phi(\iota_t^a) \right) dt + \sigma^a \widehat{dZ_t^a} \right) \tag{D.2}$$

where

$$\widehat{dZ_t^a} := dZ_t^a - \frac{\mu^O - \mu^a}{\sigma^a} dt.$$

is the Brownian motion retrieved by intermediaries. The price of the capital is given by

$$\frac{dq_t^a}{q_t^a} = \mu_t^{qa} dt + \sigma_t^{qa,a} dZ_t^a \tag{D.3}$$

where both μ_t^{qa} and $\sigma_t^{qa,a}$ are endogenous. Then, the stochastic process of capital price q_t^a perceived by experts can be written as

$$\frac{dq_t^a}{q_t^a} = \left(\mu_t^{qa} + \frac{\mu^O - \mu^a}{\sigma^a} \sigma_t^{qa,a} \right) dt + \sigma_t^{qa,a} \widehat{dZ_t^a}. \tag{D.4}$$

In contrast to our main model in Section 2, there is no technological growth and the TFP is constant. Capital k_t^a produces $y_t^a = \alpha^a k_t^a$ for intermediaries i, so the actual return process is computed as

$$dr_t^{i,ka} = \frac{d(q_t k_t^a)}{q_t k_t^a} + \underbrace{\frac{\alpha^a - \iota_t^a}{q_t^a} dt}_{\text{Dividend yield}} = \underbrace{\left(\mu_t^{qa} + \mu^a + \Phi(\iota_t^a) + \sigma^a \sigma_t^{qa,a} + \frac{\alpha^a - \iota_t^a}{q_t^a} \right)}_{\equiv r_t^{i,ka}} dt + (\sigma^a + \sigma_t^{qa,a}) dZ_t^a$$

where $\iota_t^a k_t^a$ is the amount of investment in physical capital. The agent i uses the perceived capital process (D.2) and the perceived price process (D.4) to compute the return process as

$$
\begin{aligned}
dr_t^{i,ka} &= \frac{d(q_t k_t^a)}{q_t k_t^a} + \frac{\alpha^a - \iota_t^a}{q_t^a} dt \\
&= \underbrace{\left(\mu_t^{qa} + \mu^a + \Phi(\iota_t^a) + \sigma^a \sigma_t^{qa,a} + \frac{\alpha^a - \iota_t^a}{q_t^a} + \frac{\mu^O - \mu^a}{\sigma^a}(\sigma^a + \sigma_t^{qa,a})\right)}_{\equiv \widehat{r_t^{i,ka}}} dt + (\sigma^a + \sigma_t^{qa,a})\widehat{dZ_t^a}.
\end{aligned}
$$

As in Section 2, expert i has an access to superior technology that she uses to transform capital into consumption goods at a higher rate than the households. In specific, households have $y_t^h = \ell \cdot \alpha^a k_t^h$ as their inferior production function. However, intermediaries cannot sell equity claims to the rational agent due to the market incompleteness.

The actual expected return on capital for rational household h is given by

$$
\begin{aligned}
dr_t^{h,ka} &= \frac{d(q_t^a k_t^a)}{q_t^a k_t^a} + \frac{\ell\alpha^a - \iota_t^a}{q_t^a} dt \\
&= \underbrace{\left(\mu_t^{qa} + \mu^a + \Phi(\iota_t^a) + \sigma^a \sigma_t^{qa,a} + \frac{\ell\alpha^a - \iota_t^a}{q_t^a}\right)}_{\equiv r_t^{h,ka}} dt + (\sigma^a + \sigma_t^{qa,a}) dZ_t,
\end{aligned}
$$

where $0 < \ell < 1$. The household h is rational and thus uses the correct return processes (D.1) and (D.3).

Preferences The problem of optimist i is to maximize her lifetime utility of consumption

$$
U_t^i = \sup_{C_t^i, w_t^{i,a}, \iota_t^a} \widehat{\mathbb{E}}_t \left[\int_t^\infty f(C_s^i, U_s^i) ds \right], \tag{D.5}
$$

where

$$
f(C, U) = \rho \cdot [(1-\gamma)U] \cdot \left(\log C - \frac{1}{1-\gamma} \log((1-\gamma)U) \right) \tag{D.6}
$$

is a stochastic differential utility à la Duffie and Epstein (1992) with the elasticity of intertemporal substitution fixed at one. γ is the risk aversion. We assume that intermediaries and households both have γ as their relative risk aversion. If $\gamma = 1$, (D.5) becomes a usual log-utility case. ρ is the time discount rate for both intermediaries and households. Thus, optimists maximize (D.5) subject to the budget constraint:

$$\frac{dn_t^i}{n_t^i} = \underbrace{\left(r_t + w_t^{i,a}\left(\widehat{r_t^{i,ka}} - r_t\right) - c_t^i\right)}_{\equiv \widehat{\mu_t^{ni}}} dt + \underbrace{\left(w_t^{i,a}\left(\sigma^a + \sigma_t^{qa,a}\right)\right)}_{\equiv \sigma_t^{ni,a}} \widehat{dZ_t^a}, \tag{D.7}$$

where $C_t^i = c_t^i n_t^i$ and $w_t^{i,a} := \frac{q_t^a k_t^a}{n_t^i}$ The actual dynamics with correct Brownian motion dZ_t^a will feature the mean of (D.7) as

$$\mu^{ni} = \left(1 - w_t^{i,a}\right) r_t + w_t^{ia} r_t^{i,ka} - c_t^i.$$

Solution Using the preference in (D.6), following the literature including Duffie and Epstein (1992), we guess and verify the value function as:

$$U^j(\xi_t^j, n_t^j) = \frac{(n_t^j \xi_t^j)^{1-\gamma}}{1-\gamma},$$

for both intermediaries and households, i.e., $j = i, h$, where ξ_t^j depends on the aggregate state of the economy explained later. Its dynamics is conjectured to be

$$\frac{d\xi_t^i}{\xi_t^i} = \mu_t^{\xi,i} dt + \sigma_t^{\xi i,a} dZ_t^a$$

where $\mu_t^{\xi,i}, \sigma_t^{\xi i,a}$ will be determined in equilibrium. We can write the optimist's Hamilton-Jacobi-Bellman (HJB) equation as:

$$0 = \max_{w_t^{i,a}, \iota_t^a, c_t^i} \left\{ \frac{f(c_t^i n_t^i, U_t^i)}{(\xi_t^i n_t^i)^{1-\gamma}} + \mu^{\xi i} + \widehat{\mu_t^{ni}} - \frac{\gamma}{2}(\sigma_t^{ni,a})^2 - \frac{\gamma}{2}(\sigma_t^{\xi i,a})^2 + (1-\gamma)\sigma_t^{\xi i,a}\sigma_t^{ni,a} \right\},$$

where we use $(\xi_t^i n_t^i)^{1-\gamma} = (1-\gamma)U_t^i$, which can be rewritten as

$$\begin{aligned} 0 = \max_{w_t^{i,a}, \iota_t^a, c_t^i} \Bigg\{ & \rho \log(c_t^i) - \rho \log(\xi_t^i) + \mu_t^{i,\xi} + r_t - c_t^i + w_t^{i,a}\left(\widehat{r_t^{ka}}(\iota_t^a) - r_t\right) - \frac{\gamma}{2}\left(w_t^{i,a}\right)^2 (\sigma^a + \sigma_t^{qa,a})^2 \\ & - \frac{\gamma}{2}\left(\sigma_t^{i,\xi,a}\right)^2 + (1-\gamma) w_t^{i,a} \sigma_t^{i,\xi,a} \left(\sigma^a + \sigma_t^{qa,a}\right) \Bigg\} \end{aligned}$$

The first order conditions with respect to $c_t^i, \iota_t^a, w_t^{i,a}$ are

$$c_t^i = \rho \tag{D.8}$$
$$\Phi'(\iota_t^a) = \frac{1}{q_t^a} \tag{D.9}$$
$$0 = (\widehat{r_t^{ka}} - r_t) - \gamma w_t^{i,a} (\sigma^a + \sigma_t^{qa,a})^2 + \underbrace{(1-\gamma)\sigma_t^{i,\xi,a}(\sigma^a + \sigma_t^{qa,a})}_{\text{Hedging term}}) \tag{D.10}$$

where the solution's form with $\gamma = 1$ (i.e., log-utility case) coincides with our Proposition 1. Note that with $\gamma > 1$, we have a hedging demand term as in Merton (1971).

The optimization problem and the first order conditions of agent h are identical except that as less productive household h uses the correct expectation, i.e., $r_t^{h,ka}$ is used. That is,

$$c_t^h = \rho \tag{D.11}$$
$$\Phi'(\iota_t^a) = \frac{1}{q_t^a} \tag{D.12}$$
$$0 \geq (r_t^{ka} - r_t) - \gamma w_t^{h,a} (\sigma^a + \sigma_t^{qa,a})^2 + (1-\gamma)\sigma_t^{h,\xi,a} (\sigma^a + \sigma_t^{qa,a}), \tag{D.13}$$

Market clearing We have the following market clearing conditions written in $\eta_t := \frac{n_t^i}{n_t}$, which denotes the share of wealth held by i, with the fact that both types $j = i, h$ choose the same investment ratio ι_t^a as confirmed in (D.9) and (D.12):

$$\left(c_t^i \eta_t + c_t^h (1-\eta_t)\right) q_t = (w_t^{ia}\eta_t + \ell w_t^{ha}(1-\eta_t))\alpha^a - \iota_t^a, \tag{D.14}$$

with $k_t = k_t^i + k_t^h$. The capital market clearing condition is given by

$$w_t^{ia}\eta_t + w_t^{ha}(1-\eta_t) = 1. \tag{D.15}$$

Markov equilibrium: As we do in Section 2.4, we can construct a Markov equilibrium in one state variable, η_t. We assume that each type $j = i, h$'s net worth n_t^j follows

$$\frac{dn_t^j}{n_t^j} = \mu_t^{nj} dt + \sigma_t^{nj,a} dZ_t^a,$$

where both μ_t^{nj} and $\sigma_t^{nj,a}$ are endogenous. From $\eta_t = \frac{n_t^i}{q_t^a k_t^a}$, we obtain

$$\frac{d(q_t^a k_t^a)}{q_t^a k_t^a} = \underbrace{\left(\mu^a + \Phi(\iota_t^a) + \mu_t^{qa} + \sigma^a \sigma_t^{qa,a}\right)}_{\equiv \mu_t^{qk}} dt + (\sigma^a + \sigma_t^{qa,a}) dZ_t^a,$$

which leads to

$$\begin{aligned}
\frac{d\eta_t}{\eta_t} &= \frac{dn_t^i}{n_t^i} - \frac{d(q_t^a k_t^a)}{q_t^a k_t^a} + \left(\frac{d(q_t^a k_t^a)}{q_t^a k_t^a}\right)^2 - \frac{dn_t^i}{n_t^i} \cdot \frac{d(q_t^a k_t^a)}{q_t^a k_t^a} \\
&= \left(r_t + w_t^{i,a} \left(\underbrace{r_t^{i,ka}}_{\text{True expected return}} - r \right) - c_t^i - \mu_t^{qk} + (\sigma^a + \sigma_t^{qa,a})^2 \left(1 - w_t^{i,a}\right) \right) dt + (w_t^{ia} - 1)(\sigma^a + \sigma_t^{qa,a}) dZ_t^a
\end{aligned}$$

D.1 With $\gamma = 1$ (i.e., log-utility)

With $\gamma = 1$, again, the hedging demand terms in (D.9) and (D.12) disappear. From equation (D.10), we have

$$w_t^{i,a} = \frac{\widehat{r_t^{i,ka}} - r_t}{(\sigma^a + \sigma_t^{qa,a})^2} = \frac{r_t^{i,ka} - r_t}{(\sigma^a + \sigma_t^{qa,a})^2} + \frac{\frac{\mu^O - \mu^a}{\sigma^a}(\sigma^a + \sigma_t^{qa,a})}{(\sigma^a + \sigma_t^{qa,a})^2}$$

leading to

$$r_t^{i,ka} - r_t = w_t^{i,a} (\sigma^a + \sigma_t^{qa,a})^2 - \frac{\mu^O - \mu^a}{\sigma^a} (\sigma^a + \sigma_t^{qa,a}) \tag{D.16}$$

Plugging (D.16) in the drift of η_t above, we obtain

$$\mu_t^\eta = r_t + \left(w_t^{i,a}(\sigma^a + \sigma_t^{qa,a})\right)^2 \underbrace{- w_t^{i,a} \frac{\mu^O - \mu^a}{\sigma^a} (\sigma^a + \sigma_t^{qa,a})}_{\text{Expectation error}} + (\sigma^a + \sigma_t^{qa,a})^2 (1 - w_t^{i,a}) - c_t^i - \mu_t^{qk} \tag{D.17}$$

which contains a term stemming from the expectation error of optimistic intermediaries i. For $\mu^O > \mu^a$ case, the error comes from the fact that the agent takes portfolio decisions based on a higher perceived risk premium, but gains a lower risk premium.

Note that we have

$$\begin{aligned}
r_t - \mu_t^{qk} &= \frac{\alpha^a - \iota_t^a}{q_t^a} - \left(r_t^{i,ka} - r_t\right) \\
&= \frac{\alpha^a - \iota_t^a}{q_t^a} - w_t^{i,a} (\sigma^a + \sigma_t^{qa,a})^2 + \frac{\mu^O - \mu^a}{\sigma^a} (\sigma^a + \sigma_t^{qa,a})
\end{aligned}$$

Substituting in the drift of wealth share (D.17), we obtain

$$\mu_t^\eta = \frac{\alpha^a - \iota_t^a}{q_t^a} + \left((w_t^{i,a} - 1)(\sigma^a + \sigma_t^{qa,a})\right)^2 - (w_t^{i,a} - 1)\frac{\mu^O - \mu^a}{\sigma^a}\left(\sigma^a + \sigma_t^{qa,a}\right) - \rho,$$

with

$$\sigma_t^\eta = (w_t^{i,a} - 1)(\sigma^a + \sigma_t^{qa,a}). \tag{D.18}$$

Ergodic distribution and net worth trap Again, the ratio $\tilde{D}_0 := \frac{2\mu_0^\eta}{(\sigma_0^\eta)^2}$ determines the existence of a degenerate distribution around $\eta = 0$. If $\tilde{D}_0 < 2$, the density features an infinite mass around $\eta_t = 0$. If $\tilde{D}_0 < 1$, the density becomes degenerate at $\eta_t = 0$, i.e., the economy features the net worth trap.

In our Markov equilibrium, $q_t^a = q(\eta_t)$ for some function $q(\cdot)$. Then

$$dq_t^a = q'(\eta_t)d\eta_t + \frac{1}{2}q''(\eta_t)\left(d\eta_t\right)^2,$$

leading to

$$\sigma_t^{qa,a} = \frac{q'(\eta_t)}{q(\eta_t)}\eta_t \cdot \left(w_t^{i,a} - 1\right)\left(\sigma^a + \sigma_t^{qa,a}\right), \tag{D.19}$$

where $\sigma_t^{qa,a} \to 0$ as $\eta_t \to 0$.

Around $\eta \sim 0$ At the limit $\eta_t \to 0^+$, it will be that $w_t^{h,a} \to 1$ from (D.15), from which we obtain[13]

$$q_0^a = \frac{1 + \ell\alpha^a\kappa}{1 + \rho\kappa}, \quad \iota_0^a = \frac{q_0 - 1}{\kappa} = \frac{\ell\alpha^a - \rho}{1 + \rho\kappa}, \quad \sigma_0^{qa,a} = 0,$$

leading to

$$w_0^{i,a} = \underbrace{\frac{\alpha^a}{q_0^a(\sigma^a)^2}(1 - \ell)}_{\equiv \Delta_0 > 0} + \frac{\mu^O - \mu^a}{(\sigma^a)^2} + 1 \tag{D.20}$$

$$\mu_0^\eta = \frac{\alpha^a - \iota_0}{q_0} - \rho + \left((w_0^{ia} - 1)\sigma^a\right)^2 - \frac{\mu^O - \mu^a}{\cancel{\sigma^a}}(w_0^{ia} - 1)\cancel{\sigma_a} \tag{D.21}$$

$$\sigma_0^\eta = (w_0^{ia} - 1)\sigma^a \tag{D.22}$$

[13] We put subscript 0 to denote that variables are valued at $\eta \to 0^+$ limit.

Note that due to the productivity difference, now

$$\alpha^a - \iota_0^a = \alpha^a - \frac{\ell\alpha^a - \rho}{1+\rho\kappa} = \frac{\alpha^a(1-\ell) + \rho(1+\kappa\alpha^a)}{1+\rho\kappa},$$

which with $\ell < 1$ leads to

$$\Gamma_0 \equiv \frac{\alpha^a - \iota_0^a}{q_0^a} - \rho = \frac{\alpha^a(1-\ell) + \rho(1+\kappa\alpha^a)}{1+\ell\alpha^a\kappa} - \rho > 0.$$

From equation (D.21), we obtain

$$(w_0^{i,a} - 1)^2 = \left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)^2,$$

from which we obtain

$$\mu_0^\eta = \Gamma_0 + \underbrace{\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)^2 (\sigma^a)^2}_{=(\sigma_0^\eta)^2} - (\mu^O - \mu^a)\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right), \tag{D.23}$$

$$\sigma_0^\eta = \left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)\sigma^a. \tag{D.24}$$

In order to generate a net worth trap around $\eta_t \sim 0$, we need to have $\mu_0^\eta < \frac{1}{2}(\sigma_0^\eta)^2$, i.e.,

$$\Gamma_0 + \frac{1}{2}\left(\Delta_0^2 + \cancel{\frac{2(\mu^O - \mu^a)}{(\sigma^a)^2}\Delta_0} + \frac{(\mu^O - \mu^a)^2}{(\sigma^a)^4}\right)(\sigma^a)^2 - (\mu^O - \mu^a)\left(\cancel{\Delta_0} + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right) < 0$$

leading to

$$\Gamma_0 + \frac{1}{2}\Delta_0^2(\sigma^a)^2 < \frac{(\mu^O - \mu^a)^2}{2(\sigma^a)^2}$$

Case.1 So in the optimism case ($\mu^O > \mu^a$), we need to have

$$\mu^O - \mu^a > \sqrt{2}\sigma^a\sqrt{\Gamma_0 + \frac{1}{2}(\Delta_0)^2(\sigma^a)^2} > 0$$

Case.2 In the pessimism case ($\mu^O < \mu^a$), we need to have

$$\mu^O - \mu^a < -\sqrt{2}\sigma^a\sqrt{\Gamma_0 + \frac{1}{2}(\Delta_0)^2(\sigma^a)^2} < 0.$$

Therefore, in both optimism and pessimism cases, we can have a net worth trap if the degree of optimistic or pessimistic belief of intermediaries is strong enough.

Short-sale constraint under pessimism When μ^O is low enough compared with μ^a, i.e.,

$$\mu^O - \mu^a < -(\sigma^a)^2 (1 + \Delta_0), \tag{D.25}$$

we can see from (D.20) that the optimal $w_0^{i,a}$ is negative. In that case, due to the short sale constraint of capital, the optimal $w_0^{i,a}$ around $\eta_t \sim 0$ will be $w_0^{i,a} = 0$. In that case, we obtain

$$\sigma_0^\eta = -\sigma^a, \;\; \mu_0^\eta = \Gamma_0 + (\sigma^a)^2 + \left(\mu^O - \mu^a\right).$$

Therefore, in order to have $\tilde{D}_0 < 1$, i.e., $\mu_0^\eta < \frac{1}{2}\left(\sigma_0^\eta\right)^2$, we need to have

$$\mu^O - \mu^a < -\left(\Gamma_0 + \frac{1}{2}(\sigma^a)^2\right). \tag{D.26}$$

From (D.25) and (D.26), it should be

$$\mu^O - \mu^a < -\max\left\{(\sigma^a)^2 (1 + \Delta_0), \Gamma_0 + \frac{1}{2}(\sigma^a)^2\right\}.$$

Even with mild pessimism, the sentimental agent chooses to allocate some fraction of her capital to the risky capital. This is because she has a higher productivity rate in holding capital compared to the rational agent ($l < 1$ in the model). Thus, under mild pessimism, the drift at the limit is positive since $w_0^{ia} > 1$. For excessive pessimism, $w_0^{ia} = 0$ since a higher productivity rate does not compensate the sentimental agent enough for her to take long positions in the risky capital. The no-shorting constraint forces the weight to be equal to zero. In that case, the rational household h holds all capital and earns the risk premium, which ends up draining the wealth share of the sentimental intermediary i. Figure A3 depicts that when μ^O is below some threshold, $w_0^{i,a} \geq 0$ constraint starts binding, and $\tilde{D}_0$ actually becomes negative, leading to a Dirac-delta measure at $\eta_t \sim 0$.

D.2 With $\gamma > 1$ (i.e., non log-utility)

Now, we assume $\gamma > 1$ with the elasticity of intertermporal substitution fixed at one. In this case, hedging demand terms arise, though these terms vanish as $\eta_t \to 0$ due to (C.31).

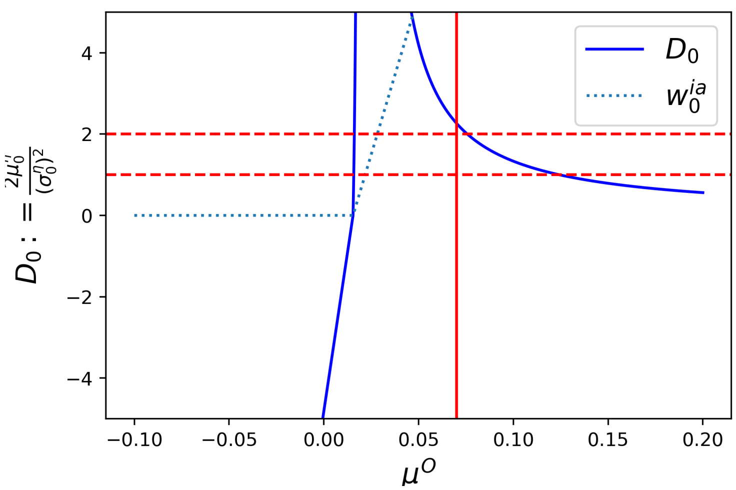


Figure A3: Net worth trap under incomplete markets.

Equation (C.30) still holds even under the incomplete market condition:

$$\begin{aligned}\mu_t^\eta =& \frac{\alpha^a - \iota_t^a}{q_t^a} + \gamma\left((w_t^{i,a} - 1)(\sigma^a + \sigma_t^{qa,a})\right)^2 - (w_t^{i,a} - 1)\frac{\mu^O - \mu^a}{\sigma^a}(\sigma^a + \sigma_t^{qa,a}) - \rho \\ &+ (1-\gamma)(\sigma^a + \sigma_t^{qa,a})^2\left(1 - w_t^{i,a}\right) + \left(1 - w_t^{i,a}\right)(1-\gamma)\underbrace{\sigma_t^{i,\xi,a}}_{\to 0 \text{ as } \eta_t \to 0}(\sigma^a + \sigma_t^{qa,a}),\end{aligned} \tag{D.27}$$

with

$$q_0^a = \frac{1 + \ell\alpha^a\kappa}{1 + \rho\kappa}, \quad \iota_0^a = \frac{q_0 - 1}{\kappa} = \frac{\ell\alpha^a - \rho}{1 + \rho\kappa}, \quad \sigma_0^{qa,a} = 0.$$

Around $\eta_t \to 0^+$, we know that $w_t^{h,a} \to 1$ from (D.15). From the household's portfolio condition (D.13), we have

$$\frac{l\alpha^a - \iota_0^a}{q_0^a} + \mu_0^{qk} - r_0 = \gamma \underbrace{w_0^{h,a}}_{\to 1}(\sigma^a)^2.$$

and from the intermediary's portfolio choice condition (D.10), we have

$$\frac{\alpha^a - \iota_0^a}{q_0^a} + \mu_0^{qk} - r_0 = \gamma w_0^{i,a}(\sigma^a)^2 - \frac{\mu^O - \mu^a}{\sigma^a}\sigma^a,$$

leading to

$$w_0^{i,a} - 1 = \frac{\mu^O - \mu^a}{\gamma(\sigma^a)^2} + \frac{(1-l)\alpha^a}{\gamma(\sigma^a)^2 q_0^a} = \frac{1}{\gamma}\left[\underbrace{\frac{(1-l)\alpha^a}{q_0^a(\sigma^a)^2}}_{\equiv\Delta_0>0} + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right].$$

Then from (D.27), we obtain

$$\begin{aligned}\mu_0^\eta = &\underbrace{\left(\frac{\alpha^a - \iota_0^a}{q_0^a} - \rho\right)}_{\equiv\Gamma_0>0} + \gamma\underbrace{\frac{1}{\gamma^2}\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)^2(\sigma^a)^2}_{=(\sigma_0^\eta)^2} - \frac{1}{\gamma}\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)\frac{\mu^O - \mu^a}{\cancel{\sigma^a}}\cancel{\sigma^a} \\ &- (1-\gamma)(\sigma^a)^2\frac{1}{\gamma}\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right),\end{aligned}$$

and

$$\sigma_0^\eta = \frac{1}{\gamma}\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)\sigma^a.$$

Therefore, the net worth trap condition, i.e., $\mu_0^\eta < \frac{1}{2}(\sigma_0^\eta)^2$, can be written as

$$\Gamma_0 - \frac{1}{\gamma}\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)\frac{\mu^O - \mu^a}{\cancel{\sigma^a}}\cancel{\sigma^a} - (1-\gamma)(\sigma^a)^2\frac{1}{\gamma}\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right) < \left(\frac{1}{2} - \gamma\right)\frac{1}{\gamma^2}\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)^2(\sigma^a)^2,$$

which can be rewritten as

$$\begin{aligned}\gamma\Gamma_0 &< \left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)\frac{\mu^O - \mu^a}{\cancel{\sigma^a}}\cancel{\sigma^a} + (1-\gamma)(\sigma^a)^2\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right) + \left(\frac{1}{2} - \gamma\right)\frac{1}{\gamma}\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)^2(\sigma^a)^2 \\ &= \left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right)\frac{\mu^O - \mu^a}{\cancel{\sigma^a}}\cancel{\sigma^a} + (1-\gamma)(\sigma^a)^2\left(\Delta_0 + \frac{\mu^O - \mu^a}{(\sigma^a)^2}\right) \\ &\quad + \left(\frac{1}{2} - \gamma\right)\frac{1}{\gamma}\left(\Delta_0^2 + \frac{2\Delta_0(\mu^O - \mu^a)}{(\sigma^a)^2} + \frac{(\mu^O - \mu^a)^2}{(\sigma^a)^4}\right)(\sigma^a)^2.\end{aligned} \tag{D.28}$$

Thus, for

$$O_1 \equiv (\gamma-1)(\gamma+\Delta_0)(\sigma^a)^2\left\{1 + \sqrt{1 + \frac{2\gamma\left\{\Delta_0(\sigma^a)^4\left[(\gamma-1) + \frac{2\gamma-1}{2\gamma}\Delta_0\right] + \gamma\Gamma_0(\sigma^a)^2\right\}}{(\gamma-1)^2(\gamma+\Delta_0)^2(\sigma^a)^4}}\right\} > 0$$

and

$$P_1 \equiv (\gamma-1)(\gamma+\Delta_0)(\sigma^a)^2 \left\{ 1 - \sqrt{1 + \frac{2\gamma\left\{\Delta_0(\sigma^a)^4\left[(\gamma-1)+\frac{2\gamma-1}{2\gamma}\Delta_0\right]+\gamma\Gamma_0(\sigma^a)^2\right\}}{(\gamma-1)^2(\gamma+\Delta_0)^2(\sigma^a)^4}} \right\} < 0,$$

Case.1 So in the optimism case ($\mu^O > \mu^a$), we need to have

$$\mu^O - \mu^a > O_1 > 0$$

Case.2 In the pessimism case ($\mu^O < \mu^a$), we need to have

$$\mu^O - \mu^a < P_1 < 0.$$

Therefore, in both optimism and pessimism cases, we can have a net worth trap if the degree of optimistic or pessimistic belief of intermediaries is strong enough.

Short-sale constraint under pessimism When μ^O is low enough compared with μ^a, i.e.,

$$\mu^O - \mu^a < -\left(\sigma^a\right)^2\left(\gamma+\Delta_0\right), \tag{D.29}$$

we can see from (B.54) that the optimal $w_0^{i,a}$ is negative. In that case, due to the short sale constraint of capital, the optimal $w_0^{i,a}$ around $\eta_t \sim 0$ will be $w_0^{i,a} = 0$. In that case, we obtain

$$\sigma_0^\eta = -\sigma^a, \;\; \mu_0^\eta = \Gamma_0 + \left(\sigma^a\right)^2 + \left(\mu^O - \mu^a\right).$$

Therefore, in order to have $\tilde{D}_0 < 1$, i.e., $\mu_0^\eta < \frac{1}{2}\left(\sigma_0^\eta\right)^2$, we need to have

$$\mu^O - \mu^a < -\left(\Gamma_0 + \frac{1}{2}\left(\sigma^a\right)^2\right). \tag{D.30}$$

From (D.29) and (D.30), it should be

$$\mu^O - \mu^a < -\max\left\{\left(\sigma^a\right)^2\left(\gamma+\Delta_0\right), \Gamma_0 + \frac{1}{2}\left(\sigma^a\right)^2\right\}.$$